LESSON MAKER PRO

# Close Enough Lines

*Use tangent lines to estimate nearby values.*

**HIGH SCHOOL · MA.912.C.3.2**

**MA.912.C.3.2** "Find an equation for the tangent line to a curve at a point and use it to make local linear approximation."

NAME _____ DATE _____

## TANGENT LINES AS ESTIMATES

At x = a, the tangent line has slope f'(a) and passes through (a, f(a)). Write the linearization as L(x) = f(a) + f'(a)(x - a). When x is close to a, use L(x) to estimate f(x).

## WORKED EXAMPLE

For $f(x) = x^2 + 5x$, find the tangent line at x = 4. Then use it to estimate f(4.2).

1. Find the derivative: f'(x) = 2x + 5.
2. At x = 4, f(4) = 36 and f'(4) = 13.
3. Write L(x) = 36 + 13(x - 4), then evaluate L(4.2) = 38.6.

**Answer: Tangent line: L(x) = 36 + 13(x - 4). Estimated value: f(4.2) ≈ 38.6.**

## PRACTICE 6 problems

**1** Find the equation of the tangent line to $f(x) = x^3 - 2x$ at x = 1.

**ANSWER** _____

**2** For h(x) = 1/x, find the tangent line at x = 2. Use the line to estimate h(2.1).

**ANSWER** _____

**3** Find the equation of the tangent line to $p(x) = x^3$ at x = -1.

**ANSWER** _____

**4** Use a tangent-line approximation for $r(x) = 2x^2 + 3$ at x = -2 to estimate r(-1.9).

**ANSWER** _____

**5** Find the equation of the tangent line to $s(x) = 4x^2 - x$ at x = 3.

**ANSWER** _____

**6** Use a tangent-line approximation for $u(x) = x^3 + x$ at x = 1 to estimate u(1.05).

**ANSWER** _____

## APPLY IT Show your work

**1** A phone's battery level, B(x), in percent, is modeled by $B(x) = 100 - 4x^2$, where x is the number of hours since the phone was fully charged. Find the tangent-line model at x = 3, then use it to estimate the battery level at x = 3.1.

WORK SPACE

**ANSWER** _____

**2** The height h(x), in meters, of a drone during a short flight is modeled by $h(x) = -x^2 + 10x$, where x is time in seconds. Find the tangent-line model at x = 4, then use it to estimate the drone's height at x = 4.1.

WORK SPACE

**ANSWER** _____

## CHALLENGE One step up

For $f(x) = x^2$, use the tangent line at x = 10 to estimate f(10.1). Then find the exact value of f(10.1) and explain why the estimate is less than the exact value.

WORK SPACE AND ANSWER

# Close Enough Lines



## PART 1 · PRACTICE

| # | PROBLEM | ANSWER |
|---|---|---|
| 1 | Find the equation of the tangent line to $f(x) = x^3 - 2x$ at $x = 1$. | **$y = x - 2$** |
| 2 | For $h(x) = 1/x$, find the tangent line at $x = 2$. Use the line to estimate $h(2.1)$. | **$L(x) = 1/2 - 1/4(x - 2)$; $h(2.1) \approx 19/40$** |
| 3 | Find the equation of the tangent line to $p(x) = x^3$ at $x = -1$. | **$y = 3x + 2$** |
| 4 | Use a tangent-line approximation for $r(x) = 2x^2 + 3$ at $x = -2$ to estimate $r(-1.9)$. | **$r(-1.9) \approx 51/5$** |
| 5 | Find the equation of the tangent line to $s(x) = 4x^2 - x$ at $x = 3$. | **$y = 23x - 36$** |
| 6 | Use a tangent-line approximation for $u(x) = x^3 + x$ at $x = 1$ to estimate $u(1.05)$. | **$u(1.05) \approx 11/5$** |

## PART 2 · APPLY IT

1. **$L(x) = 64 - 24(x - 3)$; $B(3.1) \approx 61.6\%$** $B(3) = 64$ and $B'(x) = -8x$, so $B'(3) = -24$. Substitute $x = 3.1$ into $L(x) = 64 - 24(x - 3)$.
2. **$L(x) = 24 + 2(x - 4)$; $h(4.1) \approx 24.2$ meters** $h(4) = 24$ and $h'(x) = -2x + 10$, so $h'(4) = 2$. Substitute $x = 4.1$ into $L(x) = 24 + 2(x - 4)$.

## CHALLENGE

**$L(x) = 100 + 20(x - 10)$, so $f(10.1) \approx 102$. The exact value is $f(10.1) = 102.01$. The parabola opens upward, so its tangent line at $x = 10$ lies below the curve near that point.**

Accept equivalent tangent-line equations in point-slope, slope-intercept, or expanded form. Accept equivalent fractions and terminating decimals for approximation values.