

Derivative Direction Detectives

Use derivative signs to track where a function rises or falls.

HIGH SCHOOL • MA.912.C.3.3

MA.912.C.3.3 "Determine where a function is decreasing and increasing using its derivative."

NAME _____

DATE _____

READ THE DERIVATIVE SIGN

First find $f'(x)$. Mark values in the domain where $f'(x) = 0$ or where $f'(x)$ is undefined, then test the sign of $f'(x)$ in each interval. The function is increasing where $f'(x)$ is positive and decreasing where $f'(x)$ is negative.

WORKED EXAMPLE

Determine where $h(x) = x^2 - 10x$ is increasing and decreasing.

1. Find the derivative: $h'(x) = 2x - 10$.
2. Set the derivative equal to zero: $2x - 10 = 0$, so $x = 5$.
3. For $x < 5$, $h'(x) < 0$. For $x > 5$, $h'(x) > 0$.
4. A negative derivative means decreasing, and a positive derivative means increasing.

Answer: $h(x)$ is decreasing on $(-\infty, 5)$ and increasing on $(5, \infty)$.

PRACTICE 6 problems

- 1** Determine where $f(x) = x^2 + 8x$ is increasing and decreasing.

ANSWER _____

- 2** Determine where $f(x) = -x^2 + 14x$ is increasing and decreasing.

ANSWER _____

- 3** Determine where $f(x) = x^3 - 27x$ is increasing and decreasing.

ANSWER _____

- 4** Determine where $f(x) = x^3 + 6x^2$ is increasing and decreasing.

ANSWER _____

- 5** A differentiable function has $f'(x) = (x - 1)(x + 9)$. Determine where f is increasing and decreasing.

ANSWER _____

- 6** A differentiable function has $f'(x) = -(x + 6)(x - 8)$. Determine where f is increasing and decreasing.

ANSWER _____

APPLY IT Show your work

- 1** A student club models its profit from selling batches of custom stickers by $P(x) = -x^2 + 18x - 32$, where x is the number of batches and $x \geq 0$. During what intervals is profit increasing and decreasing?

WORK SPACE

ANSWER _____

- 2** The number of active viewers for a livestream is modeled by $V(x) = -x^2 + 16x + 40$, where x is hours after the stream begins and $0 \leq x \leq 24$. During what intervals is the viewer count increasing and decreasing?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

For $f(x) = x^3/(x^2 - 1)$, determine where f is increasing and decreasing. Explain why $x = -1$ and $x = 1$ must split your intervals.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Determine where $f(x) = x^2 + 8x$ is increasing and decreasing.	Decreasing on $(-\infty, -4)$; increasing on $(-4, \infty)$.
2	Determine where $f(x) = -x^2 + 14x$ is increasing and decreasing.	Increasing on $(-\infty, 7)$; decreasing on $(7, \infty)$.
3	Determine where $f(x) = x^3 - 27x$ is increasing and decreasing.	Increasing on $(-\infty, -3)$ and $(3, \infty)$; decreasing on $(-3, 3)$.
4	Determine where $f(x) = x^3 + 6x^2$ is increasing and decreasing.	Increasing on $(-\infty, -4)$ and $(0, \infty)$; decreasing on $(-4, 0)$.
5	A differentiable function has $f'(x) = (x - 1)(x + 9)$. Determine where f is increasing and decreasing.	Increasing on $(-\infty, -9)$ and $(1, \infty)$; decreasing on $(-9, 1)$.
6	A differentiable function has $f'(x) = -(x + 6)(x - 8)$. Determine where f is increasing and decreasing.	Decreasing on $(-\infty, -6)$ and $(8, \infty)$; increasing on $(-6, 8)$.

PART 2 · APPLY IT

- Profit is increasing for $0 < x < 9$ and decreasing for $x > 9$.** $P'(x) = -2x + 18$, which is zero at $x = 9$. The derivative is positive before 9 and negative after 9 in the given domain.
- The viewer count is increasing for $0 < x < 8$ and decreasing for $8 < x < 24$.** $V'(x) = -2x + 16$, which is zero at $x = 8$. The derivative is positive from 0 to 8 and negative from 8 to 24.

CHALLENGE

$f'(x) = x^2(x^2 - 3)/(x^2 - 1)^2$. The function is increasing on $(-\infty, -\sqrt{3})$ and $(\sqrt{3}, \infty)$. It is decreasing on $(-\sqrt{3}, -1)$, $(-1, 1)$, and $(1, \sqrt{3})$. The values $x = -1$ and $x = 1$ split the intervals because f is undefined there.

Accept equivalent interval notation and sign-chart reasoning. For the challenge, accept the decreasing interval $(-1, 1)$ split at $x = 0$ into $(-1, 0)$ and $(0, 1)$, since the derivative is zero but does not change sign at $x = 0$.