LESSON MAKER PRO

Curve Clue Hunt

Use second derivatives to track how a graph bends.

HIGH SCHOOL · MA.912.C.3.5

MA.912.C.3.5 "Determine the concavity and points of inflection of a function using its second derivative."

NAME ______________________ DATE ______________

READ THE SECOND DERIVATIVE

Find f''(x), then split the number line at values where f''(x) is 0 or undefined. If f''(x) is positive, the graph is concave up. If f''(x) is negative, the graph is concave down. A point of inflection occurs only where the concavity changes and the function is defined.

WORKED EXAMPLE

Determine the concavity and point of inflection of y = x^3 + 2x^2.

1. Find the second derivative: y'' = 6x + 4.
2. Set y'' = 0: 6x + 4 = 0, so x = -2/3.
3. For x < -2/3, y'' is negative. For x > -2/3, y'' is positive.
4. Find y(-2/3) = 16/27.

Answer: Concave down on (-infinity, -2/3), concave up on (-2/3, infinity), and the point of inflection is (-2/3, 16/27).

PRACTICE

6 problems

1. Determine the concavity and point of inflection of f(x) = x^3 - 4x.

   ANSWER ______________

2. Determine the concavity and points of inflection of f(x) = x^4 - 8x^2.

   ANSWER ______________

3. Determine the concavity and point of inflection of f(x) = x^5.

   ANSWER ______________

4. Determine the concavity and points of inflection of f(x) = 1/x.

   ANSWER ______________

5. Determine the concavity and points of inflection of f(x) = x^4 + 2x^2.

   ANSWER ______________

6. Determine the concavity and points of inflection of f(x) = x^4/12 - x^3/6 - x^2.

   ANSWER ______________

APPLY IT Show your work

1 A student models the number of points earned in a study app after t hours with $P(t) = t^3 - 9t^2 + 24t$, where $0 \le t \le 6$. Determine when P is concave down and concave up. Find the point of inflection.

WORK SPACE

ANSWER ______________________

2 The height h(t), in meters, of a drone filming a school event is modeled by $h(t) = t^4/12 - t^3 + 4t^2$, where $0 \le t \le 5$. Determine the intervals where the height graph is concave up or concave down. Find each point of inflection.

WORK SPACE

ANSWER ______________________

CHALLENGE One step up

CHALLENGE

A function f is continuous for all real x, and $f''(x) = x(x - 2)^2(x + 1)$. Determine where f is concave up and concave down. List the x-values of all inflection points, and explain why $x = 2$ is not an inflection point.

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Determine the concavity and point of inflection of f(x) = x^3 - 4x.	**Concave down on (-infinity, 0), concave up on (0, infinity), and the point of inflection is (0, 0).**
2	Determine the concavity and points of inflection of f(x) = x^4 - 8x^2.	**Concave up on (-infinity, -2sqrt(3)/3) and (2sqrt(3)/3, infinity), concave down on (-2sqrt(3)/3, 2sqrt(3)/3), and the points of inflection are (-2sqrt(3)/3, -80/9) and (2sqrt(3)/3, -80/9).**
3	Determine the concavity and point of inflection of f(x) = x^5.	**Concave down on (-infinity, 0), concave up on (0, infinity), and the point of inflection is (0, 0).**
4	Determine the concavity and points of inflection of f(x) = 1/x.	**Concave down on (-infinity, 0), concave up on (0, infinity), and there are no points of inflection.**
5	Determine the concavity and points of inflection of f(x) = x^4 + 2x^2.	**Concave up on (-infinity, infinity) and there are no points of inflection.**
6	Determine the concavity and points of inflection of f(x) = x^4/12 - x^3/6 - x^2.	**Concave up on (-infinity, -1) and (2, infinity), concave down on (-1, 2), and the points of inflection are (-1, -3/4) and (2, -4).**

PART 2 · APPLY IT

1. **P is concave down on (0, 3), concave up on (3, 6), and the point of inflection is (3, 18).** P''(t) = 6t - 18. It changes from negative to positive at t = 3, and P(3) = 18.
2. **The graph is concave up on (0, 2) and (4, 5), concave down on (2, 4), and the points of inflection are (2, 28/3) and (4, 64/3).** h''(t) = t^2 - 6t + 8 = (t - 2)(t - 4). The sign changes at both values, and h(2) = 28/3 and h(4) = 64/3.

CHALLENGE

f is concave up on (-infinity, -1) and (0, infinity), and concave down on (-1, 0). The inflection x-values are x = -1 and x = 0. At x = 2, f''(x) is 0, but the factor (x - 2)^2 does not change sign, so concavity does not change.

Accept interval notation or equivalent inequality notation. For an inflection point, require evidence that concavity changes and that the function is defined at that x-value.