LESSON MAKER PRO

Curve Clues

Use second derivatives to track how a graph bends.

HIGH SCHOOL · MA.912.C.3.5

MA.912.C.3.5 "Determine the concavity and points of inflection of a function using its second derivative."

NAME _____ DATE _____

READ THE SECOND DERIVATIVE

Find f''(x), then locate where it equals 0 or does not exist. Test the sign of f'' on each interval: f''(x) > 0 means concave up, and f''(x) < 0 means concave down. A point of inflection occurs only where the concavity changes.

WORKED EXAMPLE

Determine the concavity and points of inflection of $g(x) = x^4 - 7x^2$.

1. Find derivatives: $g'(x) = 4x^3 - 14x$ and $g''(x) = 12x^2 - 14$.
2. Set $g''(x) = 0$: $12x^2 - 14 = 0$, so $x = -\text{sqrt}(7/6)$ or $x = \text{sqrt}(7/6)$.
3. Test signs: g'' is positive outside these values and negative between them.
4. Substitute $x^2 = 7/6$ into g(x): $g(x) = -245/36$.

Answer: Concave up on (-∞, -sqrt(7/6)) and (sqrt(7/6), ∞). Concave down on (-sqrt(7/6), sqrt(7/6)). Points of inflection: (-sqrt(7/6), -245/36) and (sqrt(7/6), -245/36).

PRACTICE 6 problems

1 Determine the concavity and point of inflection of $f(x) = x^3 + 5x$.

ANSWER _____

2 Determine the concavity and point of inflection of $f(x) = x^3 - 9x^2$.

ANSWER _____

3 Determine the concavity and any points of inflection of $f(x) = x^4 + 5x^2$.

ANSWER _____

4 Determine the concavity and any points of inflection of $f(x) = 1/x$.

ANSWER _____

5 Determine the concavity and any points of inflection of $f(x) = (x - 2)^4$.

ANSWER _____

6 Determine the concavity and point of inflection of $f(x) = x^3 - 3x^2$.

ANSWER _____

APPLY IT Show your work

1 A student creates a skateboard jump animation. The rider's height above a reference line, in meters, is modeled by $h(t) = t^3 - 9t^2 + 18t + 4$, where t is time in seconds and $0 \le t \le 6$. Determine when the height graph is concave up and concave down. Find the point of inflection.

WORK SPACE

ANSWER _____

2 An online game models its active users, in hundreds, by $A(t) = -t^3 + 12t^2$, where t is the number of days after a new update and $0 \le t \le 8$. Determine when the graph is concave up and concave down. Interpret the point of inflection in context.

WORK SPACE

ANSWER _____

CHALLENGE One step up

For $f(x) = x^{2/3}$, determine the concavity and decide whether the graph has a point of inflection. Explain why $x = 0$ is or is not a point of inflection.

WORK SPACE AND ANSWER

Curve Clues

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Determine the concavity and point of inflection of $f(x) = x^3 + 5x$.	**Concave down on $(-\infty, 0)$ and concave up on $(0, \infty)$. Point of inflection: (0, 0).**
2	Determine the concavity and point of inflection of $f(x) = x^3 - 9x^2$.	**Concave down on $(-\infty, 3)$ and concave up on $(3, \infty)$. Point of inflection: (3, -54).**
3	Determine the concavity and any points of inflection of $f(x) = x^4 + 5x^2$.	**Concave up on $(-\infty, \infty)$. There are no points of inflection.**
4	Determine the concavity and any points of inflection of $f(x) = 1/x$.	**Concave down on $(-\infty, 0)$ and concave up on $(0, \infty)$. There are no points of inflection because $x = 0$ is not in the domain.**
5	Determine the concavity and any points of inflection of $f(x) = (x - 2)^4$.	**Concave up on $(-\infty, 2)$ and $(2, \infty)$. There are no points of inflection because the concavity does not change at $x = 2$.**
6	Determine the concavity and point of inflection of $f(x) = x^3 - 3x^2$.	**Concave down on $(-\infty, 1)$ and concave up on $(1, \infty)$. Point of inflection: (1, -2).**

PART 2 · APPLY IT

1. **Concave down for $0 \le t < 3$ and concave up for $3 < t \le 6$. The point of inflection is (3, 4).** Compute $h''(t) = 6t - 18$. Its sign changes from negative to positive at $t = 3$, and $h(3) = 4$.
2. **Concave up for $0 \le t < 4$ and concave down for $4 < t \le 8$. The point of inflection is (4, 128), meaning that 4 days after the update, the active-user total is 12,800 and the graph changes concavity.** Compute $A''(t) = 24 - 6t$. It changes from positive to negative at $t = 4$, and $A(4) = 128$ hundreds.

CHALLENGE

$f''(x) = -2/(9x^{4/3})$ for $x \neq 0$, so the graph is concave down on $(-\infty, 0)$ and $(0, \infty)$. There is no point of inflection at $x = 0$ because f'' is negative on both sides of 0, so the concavity does not change.

Accept equivalent interval notation and equivalent exact radical forms. A point where f'' equals 0 or does not exist is an inflection point only when the concavity changes.