LESSON MAKER PRO

Derivative Graph Detective

Use f', f'' to predict a graph's shape.

HIGH SCHOOL · MA.912.C.3.6

MA.912.C.3.6 "Sketch graphs by using first and second derivatives. Compare the corresponding characteristics of the graphs of f, f' and f''."

NAME _____ DATE _____

READ THE DERIVATIVES

Where f' is positive, f rises. Where f' is negative, f falls. Where f'' is positive, f is concave up, and where f'' is negative, f is concave down. Use sign changes in f' to find extrema and sign changes in f'' to find inflection points.

WORKED EXAMPLE

Use derivatives to describe the sketch of f(x) = x^5 - 11x.

1. f'(x) = 5x^4 - 11 and f''(x) = 20x^3.
2. Set f'(x) = 0: x = ±(11/5)^(1/4).
3. f' has signs +, -, +. f'' is negative left of the y-axis and positive right of the y-axis.

Answer: f has a local maximum at x = -(11/5)^(1/4) and a local minimum at x = (11/5)^(1/4). It is concave down left of the y-axis and concave up right of the y-axis.

PRACTICE 5 problems

1 For f(x) = x^3 - 3x, list the local extrema and the concavity intervals.

ANSWER _____

2 For f(x) = x^3 + 6x, state where f is increasing and where it is concave up.

ANSWER _____

3 For f(x) = -x^3 + 3x, state where f is increasing and where it is concave down.

ANSWER _____

4 A function has f'(x) = (x - 2)(x + 1) and f''(x) = 2x - 1. State the local extrema and concavity intervals.

ANSWER _____

5 A function has f'(x) = x(x - 3)^2 and f''(x) = 3(x - 1)(x - 3). State where f increases or decreases, and list its inflection points.

ANSWER _____

APPLY IT Show your work

1 An app models its engagement score t hours after a feature launch by E(t) = t^3 - 3t^2 + 3t, for 0 <= t <= 3. Describe how the score changes and how its rate of change changes.

WORK SPACE

ANSWER _____

2 A gamer tracks net match momentum with M(t) = -t^3 + 6t^2 - 9t + 30, where t is hours into a session. Find the local extrema and describe the concavity.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

For f(x) = x^3 - 6x^2 + 9x, a student says x = 2 is a local extremum because f''(2) = 0. Is the student correct? Use f' and f'' to name the extrema and the inflection point.

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ANSWER KEY

Derivative Graph Detective

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For f(x) = x^3 - 3x, list the local extrema and the concavity intervals.	**Local maximum: (-1, 2). Local minimum: (1, -2). Concave down on x < 0 and concave up on x > 0.**
2	For f(x) = x^3 + 6x, state where f is increasing and where it is concave up.	**Increasing for all x. Concave up on x > 0.**
3	For f(x) = -x^3 + 3x, state where f is increasing and where it is concave down.	**Increasing on -1 < x < 1. Concave down on x > 0.**
4	A function has f'(x) = (x - 2)(x + 1) and f''(x) = 2x - 1. State the local extrema and concavity intervals.	**Local maximum at x = -1 and local minimum at x = 2. Concave down on x < 1/2 and concave up on x > 1/2.**
5	A function has f'(x) = x(x - 3)^2 and f''(x) = 3(x - 1)(x - 3). State where f increases or decreases, and list its inflection points.	**Decreasing on x < 0 and increasing on x > 0. Inflection points occur at x = 1 and x = 3.**

PART 2 · APPLY IT

1. **The score increases for the whole interval, with a flat tangent at t = 1. Its rate of change decreases from t = 0 to t = 1, then increases from t = 1 to t = 3.** E'(t) = 3(t - 1)^2, which is never negative. E''(t) = 6(t - 1), so concavity changes at t = 1.
2. **Local minimum: (1, 26). Local maximum: (3, 30). The graph is concave up for t < 2 and concave down for t > 2.** M'(t) = -3(t - 1)(t - 3), so the sign changes from negative to positive at 1 and positive to negative at 3. M''(t) = -6(t - 2), so concavity changes at 2.

CHALLENGE

No. f'(x) = 3(x - 1)(x - 3), so there is a local maximum at x = 1 and a local minimum at x = 3. Since f''(x) = 6(x - 2) changes sign at x = 2, x = 2 is an inflection point, not an extremum.

Accept equivalent interval notation and verbal descriptions of increasing, decreasing, and concavity. For extrema, accept the x-value alone unless the prompt asks for an ordered pair.