Peak and Valley

Use derivatives to find maximum and minimum values.

HIGH SCHOOL · MA.912.C.3.7

MA.912.C.3.7 "Solve optimization problems using derivatives."

NAME ____________________ DATE ____________

FIND THE BEST VALUE

To optimize, write a function for the quantity you want to maximize or minimize. Find where its derivative is zero, then use the function's shape, and any endpoints if given, to decide whether the value is a maximum or minimum.

WORKED EXAMPLE

Find the maximum value of f(x) = -11x^2 + 66x.

1. Find the derivative: f'(x) = -22x + 66.
2. Set the derivative equal to zero: -22x + 66 = 0, so x = 3.
3. Evaluate the function: f(3) = 99.
4. Because the x^2 coefficient is negative, this value is a maximum.

Answer: The maximum value is 99 at x = 3.

PRACTICE

6 problems

1. Find the maximum value of f(x) = -2x^2 + 16x.

ANSWER ____________________

2. Find the minimum value of g(x) = x^2 - 14x + 70.

ANSWER ____________________

3. A rectangle has a perimeter of 40 cm. Its area is A(x) = x(20 - x). Find the dimensions that maximize the area.

ANSWER ____________________

4. Find the minimum value of h(x) = 5x + 80/x, where x > 0.

ANSWER ____________________

5. For A(x) = x(30 - 2x), where 0 < x < 15, find the value of x that gives the maximum area.

ANSWER ____________________

6. A business models its profit by P(x) = -x^2 + 24x - 20. Find the value of x that gives the greatest profit and the greatest profit.

ANSWER ____________________

APPLY IT Show your work

1. The robotics club sells phone cases. At a price of \$25, it expects to sell 80 cases. For each dollar x that the club lowers the price, it expects to sell $4x$ more cases. Revenue is $R(x) = (25 - x)(80 + 4x)$. What discount gives the greatest revenue, and what are the resulting price, number of cases sold, and revenue?

WORK SPACE

ANSWER ______________________

2. For a school festival, students make a rectangular photo booth using 48 ft of rope for three sides. A wall forms the fourth side. If x is the booth's depth, what dimensions give the greatest floor area, and what is that area?

WORK SPACE

ANSWER ______________________

CHALLENGE One step up

CHALLENGE

A rectangular poster must have an area of 96 in^2. Its side lengths are x and $96/x$. What dimensions use the least border material? Explain how the derivative confirms your answer.

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the maximum value of f(x) = -2x^2 + 16x.	**Maximum value: 32 at x = 4.**
2	Find the minimum value of g(x) = x^2 - 14x + 70.	**Minimum value: 21 at x = 7.**
3	A rectangle has a perimeter of 40 cm. Its area is A(x) = x(20 - x). Find the dimensions that maximize the area.	**10 cm by 10 cm, with a maximum area of 100 cm^2.**
4	Find the minimum value of h(x) = 5x + 80/x, where x > 0.	**Minimum value: 40 at x = 4.**
5	For A(x) = x(30 - 2x), where 0 < x < 15, find the value of x that gives the maximum area.	**x = 15/2, and the maximum area is 225/2.**
6	A business models its profit by P(x) = -x^2 + 24x - 20. Find the value of x that gives the greatest profit and the greatest profit.	**Greatest profit: 124 at x = 12.**

PART 2 · APPLY IT

1. **A discount of $5/2 gives the greatest revenue. The price is $45/2, 90 cases are sold, and the greatest revenue is $2025.** Expand R(x) to get -4x^2 + 20x + 2000, then solve R'(x) = -8x + 20 = 0. This gives x = 5/2, which produces a price of $45/2 and revenue of $2025.
2. **The booth should be 12 ft deep and 24 ft along the wall. The greatest area is 288 ft^2.** The area is A(x) = x(48 - 2x), so A'(x) = 48 - 4x. Setting the derivative equal to zero gives x = 12, and the remaining side is 24.

CHALLENGE

The dimensions are 4sqrt(6) in by 4sqrt(6) in, and the least perimeter is 16sqrt(6) in. For P(x) = 2x + 192/x, P'(x) = 2 - 192/x^2 = 0 gives x^2 = 96, so x = 4sqrt(6). Also, P''(x) = 384/x^3 is positive for x > 0, confirming a minimum.

Accept equivalent exact forms and exact decimal forms, such as 22.5 for 45/2. Students may use a second derivative test, the sign of the first derivative, or the concavity of a quadratic to justify extrema.