LESSON MAKER PRO

Peak Performance

Use derivatives to find the best possible result.

HIGH SCHOOL · MA.912.C.3.7

MA.912.C.3.7 "Solve optimization problems using derivatives."

NAME _____ DATE _____

FIND THE BEST VALUE

Write the quantity you want to maximize or minimize as a function of x, and state the meaningful domain. Find critical numbers by solving f'(x) = 0 or locating where f'(x) is undefined. For an absolute maximum or minimum on a closed interval, compare the function values at critical numbers and endpoints.

WORKED EXAMPLE

Find the maximum value of $q(x) = -11x^2 + 44x + 9$.

1. Differentiate: $q'(x) = -22x + 44$.
2. Set the derivative equal to zero: $-22x + 44 = 0$, so $x = 2$.
3. Evaluate the function: $q(2) = -11(2)^2 + 44(2) + 9 = 53$.

Answer: The maximum value is 53, at x = 2.

PRACTICE 6 problems

1 Find the maximum value of $f(x) = -x^2 + 8x - 3$.

ANSWER _____

2 Find the minimum value of $g(x) = x^2 - 14x + 6$.

ANSWER _____

3 A rectangle has perimeter 30 units. Use derivatives to find the dimensions that give the greatest possible area.

ANSWER _____

4 A rectangle has its base on the x-axis and its upper vertices on $y = 18 - x^2$. Its sides are parallel to the axes. Find the maximum possible area of the rectangle.

ANSWER _____

5 Find the absolute maximum and absolute minimum of $p(x) = x^3 - 15x$ on the interval $[-4, 5]$.

ANSWER _____

6 For $x > 0$, find the value of x that minimizes $C(x) = x + 49/x$. Then state the minimum value.

ANSWER _____

APPLY IT Show your work

1 The student council models the profit from a school dance by $P(x) = -3x^2 + 90x - 300$, where x is the ticket price in dollars and $0 \le x \le 30$. What ticket price gives the greatest profit, and what is that profit?

WORK SPACE

ANSWER _____

2 The gaming club has 100 meters of fencing to make a rectangular practice area against one straight gym wall. The wall forms one side, so fencing is needed for only the other three sides. What dimensions give the greatest area, and what is the maximum area?

WORK SPACE

ANSWER _____

CHALLENGE One step up

A closed cylindrical container must hold 128pi cubic centimeters. Its radius is r centimeters and its height is h centimeters. Find the radius and height that minimize the surface area, then find the minimum surface area. Explain how you know your result is a minimum.

WORK SPACE AND ANSWER

ANSWER KEY

Peak Performance

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the maximum value of $f(x) = -x^2 + 8x - 3$.	**13, at x = 4**
2	Find the minimum value of $g(x) = x^2 - 14x + 6$.	**-43, at x = 7**
3	A rectangle has perimeter 30 units. Use derivatives to find the dimensions that give the greatest possible area.	**15/2 units by 15/2 units; maximum area = 225/4 square units**
4	A rectangle has its base on the x-axis and its upper vertices on $y = 18 - x^2$. Its sides are parallel to the axes. Find the maximum possible area of the rectangle.	**24sqrt(6) square units**
5	Find the absolute maximum and absolute minimum of $p(x) = x^3 - 15x$ on the interval [-4, 5].	**Absolute maximum: 50 at x = 5. Absolute minimum: -10sqrt(5) at x = sqrt(5).**
6	For $x > 0$, find the value of x that minimizes $C(x) = x + 49/x$. Then state the minimum value.	**x = 7; minimum value = 14**

PART 2 · APPLY IT

1. **Charge $15 per ticket. The greatest profit is $375.** Differentiate to get $P'(x) = -6x + 90$, then solve $P'(x) = 0$ to get $x = 15$. Comparing this value with the endpoints shows that $P(15) = 375$ is the greatest profit.
2. **Use a width of 25 meters and a length of 50 meters along the wall. The maximum area is 1250 square meters.** Let w be the width perpendicular to the wall, so the length is $100 - 2w$ and $A(w) = 100w - 2w^2$. Solving $A'(w) = 100 - 4w = 0$ gives $w = 25$ and length 50.

CHALLENGE

r = 4 cm, h = 8 cm, and the minimum surface area is 96pi square centimeters. Since $S(r) = 2\pi r^2 + 256\pi/r$, $S'(r) = 4\pi r - 256\pi/r^2$ is zero at r = 4. Also, $S''(r) = 4\pi + 512\pi/r^3$ is positive for r > 0, so this critical point gives a minimum.

Accept equivalent exact forms, including 225/4 for the rectangle area and radical notation written as $24\sqrt{6}$ or 24sqrt(6). For absolute-extrema items, accept correct values only when paired with the correct x-values.