

Rates in Motion

Connect secant slopes, tangent slopes, and motion.

HIGH SCHOOL • MA.912.C.3.8

MA.912.C.3.8 "Find average and instantaneous rates of change. Explain the instantaneous rate of change as the limit of the average rate of change. Interpret a derivative as a rate of change in applications, including velocity, speed and acceleration."

NAME _____

DATE _____

RATES AT A POINT

The average rate of change from $x=a$ to $x=b$ is $[f(b)-f(a)]/(b-a)$.
The instantaneous rate of change at $x=a$ is the limit of the average rates $[f(a+h)-f(a)]/h$ as h approaches 0. For position $s(t)$, velocity is $s'(t)$, speed is $|s'(t)|$, and acceleration is $v'(t)$.

WORKED EXAMPLE

Find the average rate of change of $f(x)=x^2+7$ from $x=2$ to $x=6$.

1. Find the function values: $f(2)=11$ and $f(6)=43$.
2. Use the average rate formula: $[f(6)-f(2)]/(6-2)$.
3. Compute: $(43-11)/4=32/4=8$.

Answer: 8

PRACTICE 6 problems

- 1** Find the average rate of change of $f(x)=x^2-4x$ from $x=-3$ to $x=1$.

ANSWER _____

- 2** Find the average rate of change of $g(x)=3x+5$ from $x=-4$ to $x=0$.

ANSWER _____

- 3** Find the instantaneous rate of change of $f(x)=3x^2-9x$ at $x=1$.

ANSWER _____

- 4** Find the instantaneous rate of change of $y=5x^3$ at $x=-1$.

ANSWER _____

- 5** A runner's position is $s(t)=t^2+4t$, where s is in meters and t is in seconds. Find the runner's instantaneous velocity at $t=3$.

ANSWER _____

- 6** A cart's velocity is $v(t)=12t-5$, where v is in meters per second and t is in seconds. Find its acceleration.

ANSWER _____

APPLY IT Show your work

- 1** A skateboarder's position on a straight path is $s(t)=3t^2-12t+9$, where s is in meters and t is in seconds. Find the skateboarder's instantaneous velocity and speed at $t=5$.

WORK SPACE

ANSWER _____

- 2** A student tests a small drone. Its velocity is $v(t)=4t^2-20t+21$, where v is in meters per second and t is in seconds. Find the drone's acceleration at $t=4$. Is its speed increasing or decreasing at that time?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

For $f(x)=5x^2+4x$, use $[f(-3+h)-f(-3)]/h$ to find the instantaneous rate of change at $x=-3$. Show the simplified average-rate expression, then explain in one sentence why taking h toward 0 gives the instantaneous rate.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Find the average rate of change of $f(x)=x^2-4x$ from $x=-3$ to $x=1$.	-6
2	Find the average rate of change of $g(x)=3x+5$ from $x=-4$ to $x=0$.	3
3	Find the instantaneous rate of change of $f(x)=3x^2-9x$ at $x=1$.	-3
4	Find the instantaneous rate of change of $y=5x^3$ at $x=-1$.	15
5	A runner's position is $s(t)=t^2+4t$, where s is in meters and t is in seconds. Find the runner's instantaneous velocity at $t=3$.	10 m/s
6	A cart's velocity is $v(t)=12t-5$, where v is in meters per second and t is in seconds. Find its acceleration.	12 m/s²

PART 2 • APPLY IT

- Velocity: 18 m/s. Speed: 18 m/s.** Differentiate to get $v(t)=6t-12$, then evaluate $v(5)=18$. Speed is the absolute value of velocity.
- Acceleration: 12 m/s². Its speed is increasing.** Differentiate velocity to get $a(t)=8t-20$, so $a(4)=12$. Since $v(4)=5$ and $a(4)=12$ are both positive, the speed is increasing.

CHALLENGE

Average rate = $-26+5h$; instantaneous rate = -26 . Because the average rate approaches -26 as h approaches 0, -26 is the instantaneous rate at $x=-3$.

Accept equivalent simplified forms for average rates and derivative values. For motion items, accept correct numerical answers with equivalent units, including m/s and m/s².