LESSON MAKER PRO

Rates in Motion

Connect secant slopes, tangent slopes, and motion.

HIGH SCHOOL · MA.912.C.3.8

MA.912.C.3.8 "Find average and instantaneous rates of change. Explain the instantaneous rate of change as the limit of the average rate of change. Interpret a derivative as a rate of change in applications, including velocity, speed and acceleration."

NAME _____ DATE _____

RATES AT A POINT

The average rate of change from x=a to x=b is [f(b)-f(a)]/(b-a). The instantaneous rate of change at x=a is the limit of the average rates [f(a+h)-f(a)]/h as h approaches 0. For position s(t), velocity is s'(t), speed is |s'(t)|, and acceleration is v'(t).

WORKED EXAMPLE

Find the average rate of change of $f(x)=x^2+7$ from x=2 to x=6.

1. Find the function values: f(2)=11 and f(6)=43.
2. Use the average rate formula: [f(6)-f(2)]/(6-2).
3. Compute: (43-11)/4=32/4=8.

Answer: 8

PRACTICE 6 problems

1 Find the average rate of change of $f(x)=x^2-4x$ from x=-3 to x=1.

ANSWER _____

2 Find the average rate of change of g(x)=3x+5 from x=-4 to x=0.

ANSWER _____

3 Find the instantaneous rate of change of $f(x)=3x^2-9x$ at x=1.

ANSWER _____

4 Find the instantaneous rate of change of $y=5x^3$ at x=-1.

ANSWER _____

5 A runner's position is $s(t)=t^2+4t$, where s is in meters and t is in seconds. Find the runner's instantaneous velocity at t=3.

ANSWER _____

6 A cart's velocity is v(t)=12t-5, where v is in meters per second and t is in seconds. Find its acceleration.

ANSWER _____

APPLY IT Show your work

1 A skateboarder's position on a straight path is $s(t)=3t^2-12t+9$, where s is in meters and t is in seconds. Find the skateboarder's instantaneous velocity and speed at t=5.

WORK SPACE

ANSWER _____

2 A student tests a small drone. Its velocity is $v(t)=4t^2-20t+21$, where v is in meters per second and t is in seconds. Find the drone's acceleration at t=4. Is its speed increasing or decreasing at that time?

WORK SPACE

ANSWER _____

CHALLENGE One step up

For $f(x)=5x^2+4x$, use $[f(-3+h)-f(-3)]/h$ to find the instantaneous rate of change at x=-3. Show the simplified average-rate expression, then explain in one sentence why taking h toward 0 gives the instantaneous rate.

WORK SPACE AND ANSWER

ANSWER KEY

Rates in Motion

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the average rate of change of $f(x)=x^2-4x$ from $x=-3$ to $x=1$.	**-6**
2	Find the average rate of change of $g(x)=3x+5$ from $x=-4$ to $x=0$.	**3**
3	Find the instantaneous rate of change of $f(x)=3x^2-9x$ at $x=1$.	**-3**
4	Find the instantaneous rate of change of $y=5x^3$ at $x=-1$.	**15**
5	A runner's position is $s(t)=t^2+4t$, where s is in meters and t is in seconds. Find the runner's instantaneous velocity at $t=3$.	**10 m/s**
6	A cart's velocity is $v(t)=12t-5$, where v is in meters per second and t is in seconds. Find its acceleration.	**12 m/s²**

PART 2 · APPLY IT

1. **Velocity: 18 m/s. Speed: 18 m/s.** Differentiate to get $v(t)=6t-12$, then evaluate $v(5)=18$. Speed is the absolute value of velocity.
2. **Acceleration: 12 m/s². Its speed is increasing.** Differentiate velocity to get $a(t)=8t-20$, so $a(4)=12$. Since $v(4)=5$ and $a(4)=12$ are both positive, the speed is increasing.

CHALLENGE

Average rate = -26+5h; instantaneous rate = -26. Because the average rate approaches -26 as h approaches 0, -26 is the instantaneous rate at x=-3.

Accept equivalent simplified forms for average rates and derivative values. For motion items, accept correct numerical answers with equivalent units, including m/s and m/s^2.