LESSON MAKER PRO

Peak Point

Use derivatives to locate maximums and minimums.

HIGH SCHOOL · MA.912.C.3

MA.912.C.3 "Apply derivatives to solve problems."

NAME ____________________ DATE ____________

OPTIMIZE WITH DERIVATIVES

To find where a function has a maximum or minimum, find its critical points by setting $f'(x)=0$. Use the second derivative to classify a critical point, then substitute the x-value into the original function to find the maximum or minimum value.

WORKED EXAMPLE

Find the maximum value of $p(x)=-3x^2+18x+5$.

1. Differentiate: $p'(x)=-6x+18$.
2. Set $p'(x)=0$: $-6x+18=0$, so $x=3$.
3. Since $p''(x)=-6<0$, this point gives a maximum.
4. Evaluate: $p(3)=-3(3^2)+18(3)+5=32$.

Answer: The maximum value is 32 at $x=3$.

PRACTICE 6 problems

1. For $f(x)=-2x^2+16x+1$, find the x-value and maximum value.

   ANSWER ____________________

2. For $g(x)=x^2-14x+2$, find the x-value and minimum value.

   ANSWER ____________________

3. For $h(x)=-x^2+14x-9$, find the x-value and maximum value.

   ANSWER ____________________

4. For $k(x)=2x^2-16x+9$, find the x-value and minimum value.

   ANSWER ____________________

5. A function has derivative $m'(x)=10x-40$ and has one critical point. Is the critical point a maximum or a minimum, and where is it?

   ANSWER ____________________

6. For $n(x)=-7x^2+28x+2$, find the x-value and maximum value.

   ANSWER ____________________

APPLY IT
Show your work

1 A student council tracks the number of views, in thousands, on a short video t days after it is posted. The model is $V(t)=-t^2+14t+1$ for $0 \le t \le 14$. On what day are the views greatest, and what is the greatest number of views?

WORK SPACE

ANSWER ____________________

2 A student runs a game-night snack table. Its profit P(x), in dollars, is $P(x)=-x^2+20x-4$, where x is the price, in dollars, of a snack and $0 \le x \le 20$. What price gives the greatest profit, and what is that profit?

WORK SPACE

ANSWER ____________________

CHALLENGE
One step up

CHALLENGE

On $0 \le x \le 8$, find the absolute maximum value of $q(x)=-4x^3+24x^2$ and the x-value where it occurs. Show derivative reasoning.

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For f(x)=-2x^2+16x+1, find the x-value and maximum value.	**Maximum at x=4; f(4)=33.**
2	For g(x)=x^2-14x+2, find the x-value and minimum value.	**Minimum at x=7; g(7)=-47.**
3	For h(x)=-x^2+14x-9, find the x-value and maximum value.	**Maximum at x=7; h(7)=40.**
4	For k(x)=2x^2-16x+9, find the x-value and minimum value.	**Minimum at x=4; k(4)=-23.**
5	A function has derivative m'(x)=10x-40 and has one critical point. Is the critical point a maximum or a minimum, and where is it?	**It is a minimum at x=4.**
6	For n(x)=-7x^2+28x+2, find the x-value and maximum value.	**Maximum at x=2; n(2)=30.**

PART 2 · APPLY IT

1. **The views are greatest on day 7, with 50 thousand views, or 50,000 views.** V'(t)=-2t+14=0 gives t=7. Then V(7)=50, and the critical point is a maximum because V''(t)=-2<0.
2. **A price of $10 gives the greatest profit, which is $96.** P'(x)=-2x+20=0 gives x=10. Then P(10)=96, and P''(x)=-2<0 confirms a maximum.

CHALLENGE

q'(x)=-12x^2+48x=-12x(x-4), so check x=0, x=4, and x=8. q(0)=0, q(4)=128, and q(8)=-512, so the absolute maximum is 128 at x=4.

Accept equivalent expanded or factored derivative forms. For the first word problem, accept either 50 thousand views or 50,000 views.