LESSON MAKER PRO

Rectangle Sum Lab

Estimate accumulated change with equal-width rectangles.

HIGH SCHOOL · MA.912.C.4.1

MA.912.C.4.1 "Interpret a definite integral as a limit of Riemann sums. Calculate the values of Riemann sums over equal subdivisions using left, right and midpoint evaluation points."

NAME ____________________ DATE ____________

BUILD A RIEMANN SUM

Divide the interval [a,b] into n equal parts, where Delta x = (b-a)/n. For L_n use each left endpoint, for R_n use each right endpoint, and for M_n use each midpoint, then add the rectangle areas f(x) Delta x. As n increases, these sums approach the definite integral.

WORKED EXAMPLE

Approximate the integral from 11 to 19 of x dx using the midpoint sum M_8.

1. Delta x = (19-11)/8 = 1.
2. The midpoints are 23/2, 25/2, 27/2, 29/2, 31/2, 33/2, 35/2, and 37/2.
3. M_8 = 1(23/2 + 25/2 + 27/2 + 29/2 + 31/2 + 33/2 + 35/2 + 37/2).
4. M_8 = 120.

Answer: M_8 = 120

PRACTICE 6 problems

1 Find L_4 for f(x) = x on [0,12].

ANSWER ____________________

2 Find R_3 for f(x) = x + 1 on [0,6].

ANSWER ____________________

3 Find M_2 for f(x) = x^2 on [0,4].

ANSWER ____________________

4 Find L_3 for f(x) = 2x - 1 on [1,4].

ANSWER ____________________

5 Find R_4 for f(x) = 6 - x on [0,4].

ANSWER ____________________

6 Find M_5 for f(x) = 2x + 3 on [0,10].

ANSWER ____________________

APPLY IT Show your work

1 During a four-hour gaming charity stream, the donation rate is $d(t) = 10t + 20$ dollars per hour, where t is hours after the stream begins. Use M_4 to estimate the total donations during the stream.

WORK SPACE

ANSWER ____________________

2 During the first 3 minutes of track warm-up, a student's calorie-burning rate is $c(t) = 4t + 10$ calories per minute, where t is in minutes. Use L_3 to estimate the calories burned.

WORK SPACE

ANSWER ____________________

CHALLENGE One step up

CHALLENGE

For $f(x) = 12 - 2x$ on $[0,4]$, use four equal subintervals. Find L_4 and R_4. Then give bounds for the integral from 0 to 4 of $(12 - 2x)\,dx$ and explain why the bounds work.

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find L_4 for f(x) = x on [0,12].	**L_4 = 54**
2	Find R_3 for f(x) = x + 1 on [0,6].	**R_3 = 30**
3	Find M_2 for f(x) = x^2 on [0,4].	**M_2 = 20**
4	Find L_3 for f(x) = 2x - 1 on [1,4].	**L_3 = 9**
5	Find R_4 for f(x) = 6 - x on [0,4].	**R_4 = 14**
6	Find M_5 for f(x) = 2x + 3 on [0,10].	**M_5 = 130**

PART 2 · APPLY IT

1. **$160** Each subinterval has width 1 hour, with midpoints 1/2, 3/2, 5/2, and 7/2. Add the rates 25, 35, 45, and 55, then multiply by 1 hour.
2. **42 calories** Each subinterval has width 1 minute, and the left endpoints are 0, 1, and 2. Add c(0), c(1), and c(2).

CHALLENGE

L_4 = 36 and R_4 = 28. Therefore, 28 <= integral from 0 to 4 of (12 - 2x) dx <= 36. Because f(x) is decreasing, right-endpoint rectangles underestimate and left-endpoint rectangles overestimate.

Accept equivalent arithmetic expressions and correctly labeled sums. For the challenge, accept any clear explanation that a decreasing function makes left sums overestimates and right sums underestimates.