LESSON MAKER PRO

Area Estimate Lab

Approximate accumulated change with rectangles and trapezoids.

HIGH SCHOOL · MA.912.C.4.2

MA.912.C.4.2 "Apply Riemann sums, the Trapezoidal Rule and technology to approximate definite integrals of functions represented algebraically, geometrically and by tables of values."

NAME _____ DATE _____

ESTIMATE AREA

Split the interval into equal widths, where $\Delta x = (b-a)/n$. A Riemann sum adds rectangle areas using left, right, or midpoint heights. The Trapezoidal Rule uses the average of each pair of neighboring heights, then multiplies by the width.

WORKED EXAMPLE

Use a left Riemann sum with 3 equal subintervals to approximate the integral of $f(x) = x + 4$ from $x = 1$ to $x = 7$.

1. $\Delta x = (7-1)/3 = 2$.
2. The left endpoints are 1, 3, and 5.
3. The rectangle heights are 5, 7, and 9.
4. $L_3 = 2(5 + 7 + 9) = 42$.

Answer: 42

PRACTICE 6 problems

1 Use a right Riemann sum with 4 equal subintervals to approximate the integral of $f(x) = x^2$ from $x = 0$ to $x = 4$.

ANSWER _____

2 Use the Trapezoidal Rule with 3 equal subintervals to approximate the integral of $f(x) = 2x + 1$ from $x = 0$ to $x = 3$.

ANSWER _____

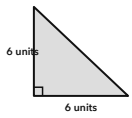
3 The values of f are $f(0) = 3$, $f(2) = 5$, $f(4) = 9$, and $f(6) = 15$. Use the Trapezoidal Rule to approximate the integral of $f(x)$ from $x = 0$ to $x = 6$.

ANSWER _____

4 Use a midpoint Riemann sum with 4 equal subintervals to approximate the integral of $f(x) = x$ from $x = 0$ to $x = 8$.

ANSWER _____

5 The shaded right triangle represents the region under a graph and above the x-axis. Find the definite integral represented by its area.



ANSWER _____

6 A right Riemann sum uses 3 rectangles, each with width 2. The rectangle heights are 4, 7, and 9. Find the sum.

ANSWER _____

APPLY IT Show your work

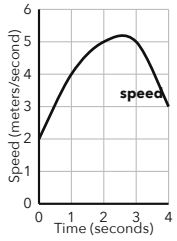
1 A concert venue tracks the rate at which people enter during the first 3 hours after doors open. Use the Trapezoidal Rule with 1-hour intervals to estimate how many people enter during those 3 hours.

VENUE ENTRY RATES

Time (hours)	Rate (people/hour)
0	18
1	26
2	30
3	34

ANSWER _____

2 A runner's speed is shown for the first 4 seconds of a workout. Use the Trapezoidal Rule with 1-second intervals to estimate the distance traveled.



ANSWER _____

CHALLENGE One step up

A spreadsheet gives equally spaced values f(0) = 2, f(1) = 3, f(2) = 6, f(3) = 11, and f(4) = 18. Find L_4, R_4, and T_4 for the integral from x = 0 to x = 4. Then explain how T_4 is related to the two Riemann sums.

WORK SPACE AND ANSWER

ANSWER KEY

Area Estimate Lab

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Use a right Riemann sum with 4 equal subintervals to approximate the integral of $f(x) = x^2$ from $x = 0$ to $x = 4$.	**30**
2	Use the Trapezoidal Rule with 3 equal subintervals to approximate the integral of $f(x) = 2x + 1$ from $x = 0$ to $x = 3$.	**12**
3	The values of f are $f(0) = 3$, $f(2) = 5$, $f(4) = 9$, and $f(6) = 15$. Use the Trapezoidal Rule to approximate the integral of $f(x)$ from $x = 0$ to $x = 6$.	**46**
4	Use a midpoint Riemann sum with 4 equal subintervals to approximate the integral of $f(x) = x$ from $x = 0$ to $x = 8$.	**32**
5	The shaded right triangle represents the region under a graph and above the x-axis. Find the definite integral represented by its area.	**18**
6	A right Riemann sum uses 3 rectangles, each with width 2. The rectangle heights are 4, 7, and 9. Find the sum.	**40**

PART 2 · APPLY IT

1. **82 people** Use $T = 1/2[18 + 2(26) + 2(30) + 34]$. The estimated total is 82 people.
2. **33/2 meters** Use the five speeds from the graph: 2, 4, 5, 5, and 3. $T = 1/2[2 + 2(4) + 2(5) + 2(5) + 3] = 33/2$ meters.

CHALLENGE

$L_4 = 22$, $R_4 = 38$, and $T_4 = 30$. T_4 is the average of L_4 and R_4.

Accept 16.5 meters for 33/2 meters. For the challenge explanation, accept any statement that $T_4 = (L_4 + R_4)/2$.