

Rate to Change

Use accumulated rate to find net change.

HIGH SCHOOL • MA.912.C.4.3

MA.912.C.4.3 "Interpret a definite integral of the rate of change of a quantity over an interval as the change of the quantity over the interval."

NAME _____

DATE _____

THE BIG IDEA

If $Q'(x)$ is the rate of change of Q , then the change in Q from $x = a$ to $x = b$ is $Q(b) - Q(a) = \int_a^b Q'(x) dx$. A positive integral means the quantity increased overall, and a negative integral means it decreased overall.

WORKED EXAMPLE

A quantity has rate of change $R'(x) = 2x + 3$ from $x = 6$ to $x = 8$. Find the change in R .

1. Change in $R = \int_6^8 (2x + 3) dx$.
2. An antiderivative of $2x + 3$ is $x^2 + 3x$.
3. $[(x^2 + 3x)]_6^8 = (64 + 24) - (36 + 18) = 34$.

Answer: The change in R is 34.

PRACTICE 6 problems

1 $A'(x) = 5$ for x from 0 to 4. Find $A(4) - A(0)$.

ANSWER _____

2 $B'(x) = x$ for x from 1 to 5. Find $B(5) - B(1)$.

ANSWER _____

3 $C'(x) = 7 - x$ for x from 0 to 4. Find $C(4) - C(0)$.

ANSWER _____

4 $D'(x) = 4x + 1$ for x from 0 to 5. Find $D(5) - D(0)$.

ANSWER _____

5 $E'(x) = x^2$ for x from 1 to 4. Find $E(4) - E(1)$.

ANSWER _____

6 $F'(x) = x - 5$ for x from 1 to 5. Find $F(5) - F(1)$.

ANSWER _____

APPLY IT Show your work

- 1** A gaming channel's video gets new views at a rate of $V'(x) = 5x + 4$ views per minute, where x is the number of minutes after posting. How many new views does the model predict from $x = 0$ to $x = 4$?

WORK SPACE

ANSWER _____

- 2** A phone battery is charging at a rate of $B'(x) = 12 - x$ percentage points per hour, where x is hours since charging began. Find the change in battery charge from $x = 1$ to $x = 5$.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

At $x = 0$, a music festival app has 30 downloads. Its download rate is $D'(x) = x^2 - 4x + 5$ downloads per hour for $0 \leq x \leq 4$. Find $D(4)$. Then write one sentence explaining why $D'(4)$ is not the number of downloads added over the four hours.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	$A'(x) = 5$ for x from 0 to 4. Find $A(4) - A(0)$.	20
2	$B'(x) = x$ for x from 1 to 5. Find $B(5) - B(1)$.	12
3	$C'(x) = 7 - x$ for x from 0 to 4. Find $C(4) - C(0)$.	20
4	$D'(x) = 4x + 1$ for x from 0 to 5. Find $D(5) - D(0)$.	55
5	$E'(x) = x^2$ for x from 1 to 4. Find $E(4) - E(1)$.	21
6	$F'(x) = x - 5$ for x from 1 to 5. Find $F(5) - F(1)$.	-8

PART 2 · APPLY IT

- 56 new views** Compute $\int_0^4 (5x + 4) dx = [(5/2)x^2 + 4x]_0^4 = 56$.
- 36 percentage points** Compute $\int_1^5 (12 - x) dx = [12x - x^2/2]_1^5 = 36$.

CHALLENGE

$D(4) = 118/3$ downloads. $D'(4) = 5$ downloads per hour is the instantaneous rate at $x = 4$, while the total added downloads are found by integrating the rate from 0 to 4.

Accept equivalent exact forms, including 36 percentage points for the battery problem. For the challenge, require both the integrated total and a statement that distinguishes instantaneous rate from change over an interval.