LESSON MAKER PRO

Rate to Change

Use accumulated rate to find net change.

HIGH SCHOOL · MA.912.C.4.3

MA.912.C.4.3 "Interpret a definite integral of the rate of change of a quantity over an interval as the change of the quantity over the interval."

NAME _____ DATE _____

THE BIG IDEA

If Q'(x) is the rate of change of Q, then the change in Q from x = a to x = b is Q(b) - Q(a) = ∫_a^b Q'(x) dx. A positive integral means the quantity increased overall, and a negative integral means it decreased overall.

WORKED EXAMPLE

A quantity has rate of change R'(x) = 2x + 3 from x = 6 to x = 8. Find the change in R.

1. Change in R = ∫_6^8 (2x + 3) dx.
2. An antiderivative of 2x + 3 is x^2 + 3x.
3. [(x^2 + 3x)]_6^8 = (64 + 24) - (36 + 18) = 34.

Answer: The change in R is 34.

PRACTICE 6 problems

1 A'(x) = 5 for x from 0 to 4. Find A(4) - A(0).

ANSWER _____

2 B'(x) = x for x from 1 to 5. Find B(5) - B(1).

ANSWER _____

3 C'(x) = 7 - x for x from 0 to 4. Find C(4) - C(0).

ANSWER _____

4 D'(x) = 4x + 1 for x from 0 to 5. Find D(5) - D(0).

ANSWER _____

5 E'(x) = x^2 for x from 1 to 4. Find E(4) - E(1).

ANSWER _____

6 F'(x) = x - 5 for x from 1 to 5. Find F(5) - F(1).

ANSWER _____

APPLY IT Show your work

1 A gaming channel's video gets new views at a rate of V'(x) = 5x + 4 views per minute, where x is the number of minutes after posting. How many new views does the model predict from x = 0 to x = 4?

WORK SPACE

ANSWER _____

2 A phone battery is charging at a rate of B'(x) = 12 - x percentage points per hour, where x is hours since charging began. Find the change in battery charge from x = 1 to x = 5.

WORK SPACE

ANSWER _____

CHALLENGE One step up

At x = 0, a music festival app has 30 downloads. Its download rate is D'(x) = x^2 - 4x + 5 downloads per hour for 0 <= x <= 4. Find D(4). Then write one sentence explaining why D'(4) is not the number of downloads added over the four hours.

WORK SPACE AND ANSWER

ANSWER KEY # Rate to Change

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	A'(x) = 5 for x from 0 to 4. Find A(4) - A(0).	**20**
2	B'(x) = x for x from 1 to 5. Find B(5) - B(1).	**12**
3	C'(x) = 7 - x for x from 0 to 4. Find C(4) - C(0).	**20**
4	D'(x) = 4x + 1 for x from 0 to 5. Find D(5) - D(0).	**55**
5	E'(x) = x^2 for x from 1 to 4. Find E(4) - E(1).	**21**
6	F'(x) = x - 5 for x from 1 to 5. Find F(5) - F(1).	**-8**

PART 2 · APPLY IT

1. **56 new views** Compute ∫_0^4 (5x + 4) dx = [(5/2)x^2 + 4x]_0^4 = 56.
2. **36 percentage points** Compute ∫_1^5 (12 - x) dx = [12x - x^2/2]_1^5 = 36.

CHALLENGE

D(4) = 118/3 downloads. D'(4) = 5 downloads per hour is the instantaneous rate at x = 4, while the total added downloads are found by integrating the rate from 0 to 4.

Accept equivalent exact forms, including 36 percentage points for the battery problem. For the challenge, require both the integrated total and a statement that distinguishes instantaneous rate from change over an interval.