

Area Tells Change

Use derivative graphs and accumulation to track function values.

HIGH SCHOOL • MA.912.C.4.5

MA.912.C.4.5 "Analyze function graphs by using derivative graphs and the Fundamental Theorem of Calculus."

NAME _____

DATE _____

CHANGE FROM AREA

If $A(x) = \int_a^x f(t) dt$, then $A'(x) = f(x)$. The signed area under a derivative graph gives the change in the original function, so $f(b) - f(a) = \int_a^b f'(x) dx$.

WORKED EXAMPLE

Let $A(x) = \int_{-8}^x (t+5) dt$. Find $A(10)$.

- $A(10) = \int_{-8}^{10} (t+5) dt$.
- An antiderivative of $t+5$ is $t^2/2 + 5t$.
- $A(10) = [t^2/2 + 5t]_{-8}^{10}$.
- $A(10) = (50 + 50) - (32 + 40) = 28$.

Answer: 28

PRACTICE 6 problems

- 1** Let $P(x) = \int_{-1}^x (2t+3) dt$. Find $P(4)$.

ANSWER _____

- 2** Let $Q(x) = \int_0^x (t^2-2) dt$. Find $Q'(3)$.

ANSWER _____

- 3** The graph of $r'(x)$ is horizontal at $y=6$ from $x=1$ to $x=3$. Find $r(3) - r(1)$.

ANSWER _____

- 4** The graph of $s'(x)$ is horizontal at $y=-2$ from $x=-4$ to $x=-1$. Find $s(-1) - s(-4)$.

ANSWER _____

- 5** The graph of $h'(x)$ is a line segment from $(0, 3)$ to $(4, 0)$. If $h(0)=1$, find $h(4)$.

ANSWER _____

- 6** Let $g(x) = \int_{-2}^x (3-t) dt$. On which part of $-2 \leq x \leq 4$ is g increasing?

ANSWER _____

APPLY IT Show your work

- 1** A robotics team models its battery charge $B(t)$, in percent, during a test. The graph of $B'(t)$, in percentage points per minute, is a line segment from $(0, -3)$ to $(4, 1)$. If $B(0)=70$, find $B(4)$.

WORK SPACE

ANSWER _____

- 2** A student council posts concert clips online. The follower rate is $F'(t)=2t+1$ followers per day for $0 \leq t \leq 3$. At $t=0$, the account has 40 followers. How many followers does it have at $t=3$?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

The graph of $p'(x)$ consists of line segments joining $(-4, -2)$, $(0, 2)$, $(3, -1)$, and $(6, 2)$. If $p(-4)=12$, find $p(6)$ and all x -values where p has a local maximum or local minimum.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Let $P(x) = \int_{-1}^x (2t+3) dt$. Find $P(4)$.	24
2	Let $Q(x) = \int_0^x (t^2-2) dt$. Find $Q'(3)$.	7
3	The graph of $r'(x)$ is horizontal at $y=6$ from $x=1$ to $x=3$. Find $r(3)-r(1)$.	12
4	The graph of $s'(x)$ is horizontal at $y=-2$ from $x=-4$ to $x=-1$. Find $s(-1)-s(-4)$.	-6
5	The graph of $h'(x)$ is a line segment from $(0, 3)$ to $(4, 0)$. If $h(0)=1$, find $h(4)$.	7
6	Let $g(x) = \int_{-2}^x (3-t) dt$. On which part of $2 \leq x \leq 4$ is g increasing?	$2 \leq x \leq 3$

PART 2 • APPLY IT

- 66%** Find the signed area under $B'(t)$ from 0 to 4. The area is -4 percentage points, so subtract 4 from 70.
- 52 followers** Integrate $2t+1$ from 0 to 3 to find the increase in followers. Add that increase to the starting total of 40.

CHALLENGE

$p(6)=15$; local minima at $x=-2$ and $x=4$; local maximum at $x=2$

Accept $[2, 3]$ as equivalent notation for problem 6. For graph-based items, students may use signed rectangles, triangles, or trapezoids to find the same net change.