

# Integral Property Quest

*Use integral rules to combine, split, and reverse intervals.*

**HIGH SCHOOL • MA.912.C.4.6**

**MA.912.C.4.6** "Evaluate or solve problems using the properties of definite integrals. Properties are limited to the following:"

NAME \_\_\_\_\_

DATE \_\_\_\_\_

## INTEGRAL RULES

An integral over a whole interval equals the sum of integrals over adjoining intervals. Reversing the bounds changes the sign, and an integral can distribute across sums and constant multiples. The integral from a value to itself is zero.

## WORKED EXAMPLE

Given integral from 2 to 6 of  $f(x)$   $dx = 9$  and integral from 2 to 6 of  $g(x)$   $dx = -3$ , evaluate integral from 2 to 6 of  $[2f(x) - g(x)]$   $dx$ .

1. Distribute the integral:  $2(\text{integral from 2 to 6 of } f(x) \text{ } dx) - \text{integral from 2 to 6 of } g(x) \text{ } dx$ .
2. Substitute the known values:  $2(9) - (-3)$ .
3. Simplify:  $18 + 3 = 21$ .

**Answer: 21**

## PRACTICE 6 problems

- 1** Given integral from 1 to 4 of  $p(x)$   $dx = 7$ , find integral from 4 to 1 of  $p(x)$   $dx$ .

**ANSWER** \_\_\_\_\_

- 2** Given integral from -5 to 1 of  $q(x)$   $dx = -8$ , find integral from -5 to -5 of  $q(x)$   $dx$ .

**ANSWER** \_\_\_\_\_

- 3** Given integral from 0 to 4 of  $r(x)$   $dx = 5$  and integral from 4 to 7 of  $r(x)$   $dx = -1$ , find integral from 0 to 7 of  $r(x)$   $dx$ .

**ANSWER** \_\_\_\_\_

- 4** Given integral from -4 to 5 of  $s(x)$   $dx = 12$  and integral from -4 to 1 of  $s(x)$   $dx = 8$ , find integral from 1 to 5 of  $s(x)$   $dx$ .

**ANSWER** \_\_\_\_\_

- 5** Given integral from 4 to 8 of  $h(x)$   $dx = -7$ , find integral from 4 to 8 of  $-5h(x)$   $dx$ .

**ANSWER** \_\_\_\_\_

- 6** Given integral from -8 to -1 of  $u(x)$   $dx = 15$  and integral from -8 to -1 of  $v(x)$   $dx = -4$ , find integral from -8 to -1 of  $[u(x) + v(x)]$   $dx$ .

**ANSWER** \_\_\_\_\_

## APPLY IT Show your work

- 1** During a livestream,  $r(t)$  is the rate of change in viewers, in viewers per minute. The stream has 240 viewers at  $t = 0$ . Given  $\int_0^5 r(t) dt = 18$  and  $\int_5^{11} r(t) dt = -7$ , how many viewers are watching at  $t = 11$ ?

WORK SPACE

ANSWER \_\_\_\_\_

- 2** A student's phone battery changes at rate  $b(t)$ , measured in percentage points per hour. At  $t = 1$ , the battery is 34%. Given  $\int_7^1 b(t) dt = -16$ , what is the battery percentage at  $t = 7$ ?

WORK SPACE

ANSWER \_\_\_\_\_

## CHALLENGE One step up

### CHALLENGE

Given  $\int_{-7}^{-1} f(x) dx = 4$ ,  $\int_{-1}^5 f(x) dx = 13$ , and  $\int_{-7}^5 g(x) dx = -8$ , evaluate  $\int_5^{-7} [4f(x) + 5g(x)] dx$ .

---

---

## PART 1 • PRACTICE

| # | PROBLEM                                                                                                                                           | ANSWER    |
|---|---------------------------------------------------------------------------------------------------------------------------------------------------|-----------|
| 1 | Given integral from 1 to 4 of $p(x) dx = 7$ , find integral from 4 to 1 of $p(x) dx$ .                                                            | <b>-7</b> |
| 2 | Given integral from -5 to 1 of $q(x) dx = -8$ , find integral from -5 to -5 of $q(x) dx$ .                                                        | <b>0</b>  |
| 3 | Given integral from 0 to 4 of $r(x) dx = 5$ and integral from 4 to 7 of $r(x) dx = -1$ , find integral from 0 to 7 of $r(x) dx$ .                 | <b>4</b>  |
| 4 | Given integral from -4 to 5 of $s(x) dx = 12$ and integral from -4 to 1 of $s(x) dx = 8$ , find integral from 1 to 5 of $s(x) dx$ .               | <b>4</b>  |
| 5 | Given integral from 4 to 8 of $h(x) dx = -7$ , find integral from 4 to 8 of $-5h(x) dx$ .                                                         | <b>35</b> |
| 6 | Given integral from -8 to -1 of $u(x) dx = 15$ and integral from -8 to -1 of $v(x) dx = -4$ , find integral from -8 to -1 of $[u(x) + v(x)] dx$ . | <b>11</b> |

## PART 2 • APPLY IT

- 251 viewers** Combine the adjoining intervals to get a net viewer change of  $18 + (-7) = 11$ . Add 11 to the starting total of 240.
- 50%** Reverse the bounds, so integral from 1 to 7 of  $b(t) dt = 16$ . Add 16 percentage points to 34%.

## CHALLENGE

**-28**

Accept equivalent notation for definite integrals and correct intermediate expressions. For the word problems, accept answers that include the correct numerical result with appropriate units.