LESSON MAKER PRO

Inside Changeup

Use substitution to turn complex integrals into familiar ones.

HIGH SCHOOL · MA.912.C.4.7

MA.912.C.4.7 "Evaluate definite and indefinite integrals by using integration by substitution."

NAME _____ DATE _____

MATCH THE INSIDE

Choose an inside expression whose derivative, or a constant multiple of its derivative, also appears in the integrand. Replace the inside expression and its derivative together, integrate using the power rule or the natural log rule, then substitute back. For a definite integral, change the bounds to values of the inside expression or substitute back before using the original bounds.

WORKED EXAMPLE

Evaluate $\int 14x(7x^2 + 3)^8\,dx$.

1. Choose the inside expression $7x^2 + 3$. Its derivative is $14x$.
2. Since $14x\,dx = d(7x^2 + 3)$, rewrite the integral as $\int (7x^2 + 3)^8\,d(7x^2 + 3)$.
3. Apply the power rule: $(7x^2 + 3)^9/9 + C$.

Answer: $(7x^2 + 3)^9/9 + C$

PRACTICE 6 problems

1 Evaluate $\int 5x^4(x^5 + 6)^{10}\,dx$.

ANSWER _____

2 Evaluate $\int 6x^5/(x^6 + 4)\,dx$.

ANSWER _____

3 Evaluate $\int_0^1 5x^4(x^5 + 4)^{10}\,dx$.

ANSWER _____

4 Evaluate $\int_0^4 4/(4x + 5)^6\,dx$.

ANSWER _____

5 Evaluate $\int 20/(4x - 5)\,dx$.

ANSWER _____

6 Evaluate $\int 5x^4/(x^5 + 1)^6\,dx$.

ANSWER _____

APPLY IT Show your work

1 At x hours after a livestream begins, a music app awards bonus credits at a rate of $6x^5(x^6 + 5)^4$ credits per hour. How many credits does the app award from x = 0 to x = 1?

WORK SPACE

ANSWER _____

2 At x days after a video is posted, a student channel's view rate is $11x^{10}/(x^{11} + 4)$ thousand views per day. How many thousand views does the model predict during the first day?

WORK SPACE

ANSWER _____

CHALLENGE One step up

Evaluate $\int_0^1 20x^4/(x^5 + 5)^6\,dx$. Show a substitution and keep the answer exact.

WORK SPACE AND ANSWER

ANSWER KEY

Inside Changeup

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Evaluate $\int 5x^4(x^5 + 6)^{10}\,dx$.	$\mathbf{(x^5 + 6)^{11}/11 + C}$
2	Evaluate $\int 6x^5/(x^6 + 4)\,dx$.	$\mathbf{\ln\lvert x^6 + 4\rvert + C}$
3	Evaluate $\int_0^1 5x^4(x^5 + 4)^{10}\,dx$.	$\mathbf{(5^{11} - 4^{11})/11}$
4	Evaluate $\int_0^4 4/(4x + 5)^6\,dx$.	$\mathbf{(1/5)(1/5^5 - 1/21^5)}$
5	Evaluate $\int 20/(4x - 5)\,dx$.	$\mathbf{5\ln\lvert 4x - 5\rvert + C}$
6	Evaluate $\int 5x^4/(x^5 + 1)^6\,dx$.	$\mathbf{-1/(5(x^5 + 1)^5) + C}$

PART 2 · APPLY IT

1. **4651/5 credits** Use the inside expression $x^6 + 5$, whose derivative is $6x^5$. Evaluate $(x^6 + 5)^5/5$ from $x = 0$ to $x = 1$.
2. **ln(5/4) thousand views** Use the inside expression $x^{11} + 4$, whose derivative is $11x^{10}$. The bounds change from 0 and 1 to 4 and 5, so evaluate $\ln(x^{11} + 4)$.

CHALLENGE

$\mathbf{(4/5)(1/5^5 - 1/6^5)}$

Accept algebraically equivalent exact forms for all answers. For the logarithm answers, absolute value bars are required for $5\ln|4x - 5|$, while $\ln(x^6 + 4)$ is equivalent because $x^6 + 4$ is always positive.