

Integral Accumulation

Use antiderivatives to find exact accumulated change.

HIGH SCHOOL • MA.912.C.4

MA.912.C.4 "Develop an understanding for and determine integrals."

NAME _____

DATE _____

EVALUATE A DEFINITE INTEGRAL

A definite integral adds up a rate of change over an interval. Find an antiderivative $F(x)$, then subtract the lower value from the upper value: $F(b) - F(a)$.

WORKED EXAMPLE

Evaluate $\int_{-1}^0 13x^{12} dx$.

1. An antiderivative of $13x^{12}$ is x^{13} .
2. Apply the bounds: $[x^{13}]_{-1}^0 = 0^{13} - (-1)^{13}$.
3. $0 - (-1) = 1$.

Answer: 1

PRACTICE 6 problems

1 Evaluate $\int_{-3}^4 5x^3 dx$.

ANSWER _____

2 Evaluate $\int_{-4}^5 (6x^4 - 9) dx$.

ANSWER _____

3 Evaluate $\int_{-3}^6 5x dx$.

ANSWER _____

4 Evaluate $\int_{-4}^{-2} (x^3 + 10) dx$.

ANSWER _____

5 Evaluate $\int_{-5}^9 (14 - x) dx$.

ANSWER _____

6 Evaluate $\int_{-4}^9 3\sqrt{x} dx$.

ANSWER _____

APPLY IT Show your work

- 1** A robotics club tests a delivery robot. Its velocity is $v(t) = 6t + 14$ meters per minute from $t = 3$ to $t = 5$ minutes. How far does the robot travel during this interval?

WORK SPACE

ANSWER _____

- 2** At a school concert, the rate of ticket sales is $r(t) = 4t^2 + 9$ tickets per hour from $t = 3$ to $t = 6$ hours after sales open. How many tickets are sold during this interval?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Evaluate $\int_{-6}^4 |x + 3| dx$. Show how you split the integral at the point where the expression inside the absolute value equals zero.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Evaluate $\int_{-3}^4 5x^3 \, dx$.	875/4
2	Evaluate $\int_{-4}^5 (6x^4 - 9) \, dx$.	12561/5
3	Evaluate $\int_{-3}^6 5x \, dx$.	135/2
4	Evaluate $\int_{-4}^{-2} (x^3 + 10) \, dx$.	-40
5	Evaluate $\int_{-5}^9 (14 - x) \, dx$.	28
6	Evaluate $\int_{-4}^9 3\sqrt{x} \, dx$.	38

PART 2 • APPLY IT

1. **76 meters** Integrate the positive velocity: $\int_{-3}^5 (6t + 14) \, dt$. Evaluate $[3t^2 + 14t]_{-3}^5$ to get 76.
2. **279 tickets** Find accumulated tickets with $\int_{-3}^6 (4t^2 + 9) \, dt$. Evaluate $[(4/3)t^3 + 9t]_{-3}^6$ to get 279.

CHALLENGE

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Accept equivalent exact fraction forms for the definite integrals. For the challenge, students should split at $x = -3$ and may use geometric areas or piecewise antiderivatives.