

Separate and Solve

Use integration to solve differential equations with initial values.

HIGH SCHOOL • MA.912.C.5.2

MA.912.C.5.2 "Solve separable differential equations."

NAME _____

DATE _____

SEPARATE, INTEGRATE, SUBSTITUTE

When dy/dx is a product of an x -expression and a y -expression, move the y -expression with dy and the x -expression with dx . Integrate both sides, then use the initial value to find the constant. Write the solution so it matches the given starting value.

WORKED EXAMPLE

Solve $dy/dx = 13y$, given $y(8) = 11$.

- $dy/y = 13 dx$
- $\ln|y| = 13x + C$
- $y = Ce^{13x}$, and $11 = Ce^{104}$, so $C = 11e^{-104}$
- Substitute C into the equation.

Answer: $y = 11e^{13x - 104}$

PRACTICE 6 problems

1 Solve $dy/dx = 2xy$, given $y(0) = 5$.

ANSWER _____

2 Solve $dy/dx = 3x^2y$, given $y(0) = 4$.

ANSWER _____

3 Solve $dy/dx = 6x/y$, given $y(0) = 1$.

ANSWER _____

4 Solve $dy/dx = (x + 1)y$, given $y(0) = 2$.

ANSWER _____

5 Solve $dy/dx = 9x/y^3$, given $y(0) = 3$.

ANSWER _____

6 Solve $dy/dx = y/(x + 2)$, given $y(0) = 3$.

ANSWER _____

APPLY IT Show your work

- 1** A phone's stored charge $B(t)$, measured in charge units, changes while charging according to $dB/dt = 12t/B$. At $t = 0$ hours, $B = 40$. Find $B(t)$ for $t \geq 0$.

WORK SPACE

ANSWER _____

- 2** The number of online sign-ups $N(t)$ for a gaming club is modeled by $dN/dt = -N/3$, where t is measured in days. At $t = 0$, there are 18 sign-ups. Find $N(t)$.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Solve $dy/dx = x(y + 1)^2$, given $y(0) = 0$. State the largest interval containing 0 where your solution is defined.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Solve $dy/dx = 2xy$, given $y(0) = 5$.	$y = 5e^{(x^2)}$
2	Solve $dy/dx = 3x^2y$, given $y(0) = 4$.	$y = 4e^{(x^3)}$
3	Solve $dy/dx = 6x/y$, given $y(0) = 1$.	$y = \sqrt{6x^2 + 1}$
4	Solve $dy/dx = (x + 1)y$, given $y(0) = 2$.	$y = 2e^{(x^2/2 + x)}$
5	Solve $dy/dx = 9x/y^3$, given $y(0) = 3$.	$y = (18x^2 + 81)^{1/4}$
6	Solve $dy/dx = y/(x + 2)$, given $y(0) = 3$.	$y = 3(x + 2)/2$, for $x > -2$

PART 2 · APPLY IT

1. **$B(t) = \sqrt{12t^2 + 1600}$** Separate to get $B \, dB = 12t \, dt$, then integrate. Use $B(0) = 40$ and choose the positive square root because stored charge is positive.
2. **$N(t) = 18e^{(-t/3)}$** Separate to get $dN/N = -dt/3$, then integrate. Use $N(0) = 18$ to determine the constant.

CHALLENGE

$y = x^2/(2 - x^2)$, for $-\sqrt{2} < x < \sqrt{2}$

Accept algebraically equivalent exponential forms. For the square-root and fourth-root solutions, accept only the positive branch that matches the initial value; include the stated domain for problem 6 and the challenge.