LESSON MAKER PRO

Separate and Solve

Use integration to solve differential equations with initial values.

HIGH SCHOOL · MA.912.C.5.2

MA.912.C.5.2 "Solve separable differential equations."

NAME _____ DATE _____

SEPARATE, INTEGRATE, SUBSTITUTE

When dy/dx is a product of an x-expression and a y-expression, move the y-expression with dy and the x-expression with dx. Integrate both sides, then use the initial value to find the constant. Write the solution so it matches the given starting value.

WORKED EXAMPLE

Solve $dy/dx = 13y$, given $y(8) = 11$.

1. $dy/y = 13\ dx$
2. $\ln|y| = 13x + C$
3. $y = Ce^{(13x)}$, and $11 = Ce^{104}$, so $C = 11e^{(-104)}$
4. Substitute C into the equation.

Answer: $y = 11e^{(13x - 104)}$

PRACTICE 6 problems

1 Solve $dy/dx = 2xy$, given $y(0) = 5$.

ANSWER _____

2 Solve $dy/dx = 3x^2y$, given $y(0) = 4$.

ANSWER _____

3 Solve $dy/dx = 6x/y$, given $y(0) = 1$.

ANSWER _____

4 Solve $dy/dx = (x + 1)y$, given $y(0) = 2$.

ANSWER _____

5 Solve $dy/dx = 9x/y^3$, given $y(0) = 3$.

ANSWER _____

6 Solve $dy/dx = y/(x + 2)$, given $y(0) = 3$.

ANSWER _____

APPLY IT Show your work

1 A phone's stored charge B(t), measured in charge units, changes while charging according to dB/dt = 12t/B. At t = 0 hours, B = 40. Find B(t) for t >= 0.

WORK SPACE

ANSWER _____

2 The number of online sign-ups N(t) for a gaming club is modeled by dN/dt = -N/3, where t is measured in days. At t = 0, there are 18 sign-ups. Find N(t).

WORK SPACE

ANSWER _____

CHALLENGE One step up

Solve $dy/dx = x(y + 1)^2$, given y(0) = 0. State the largest interval containing 0 where your solution is defined.

WORK SPACE AND ANSWER

ANSWER KEY # Separate and Solve

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Solve dy/dx = 2xy, given y(0) = 5.	**$y = 5e^{(x^2)}$**
2	Solve dy/dx = $3x^2y$, given y(0) = 4.	**$y = 4e^{(x^3)}$**
3	Solve dy/dx = 6x/y, given y(0) = 1.	**$y = \text{sqrt}(6x^2 + 1)$**
4	Solve dy/dx = (x + 1)y, given y(0) = 2.	**$y = 2e^{(x^2/2 + x)}$**
5	Solve dy/dx = $9x/y^3$, given y(0) = 3.	**$y = (18x^2 + 81)^{(1/4)}$**
6	Solve dy/dx = y/(x + 2), given y(0) = 3.	**y = 3(x + 2)/2, for x > -2**

PART 2 · APPLY IT

1. **$B(t) = \text{sqrt}(12t^2 + 1600)$** Separate to get B dB = 12t dt, then integrate. Use B(0) = 40 and choose the positive square root because stored charge is positive.
2. **$N(t) = 18e^{(-t/3)}$** Separate to get dN/N = -dt/3, then integrate. Use N(0) = 18 to determine the constant.

CHALLENGE

$y = x^2/(2 - x^2)$, for $-\text{sqrt}(2) < x < \text{sqrt}(2)$

Accept algebraically equivalent exponential forms. For the square-root and fourth-root solutions, accept only the positive branch that matches the initial value; include the stated domain for problem 6 and the challenge.