LESSON MAKER PRO

Change Over Time

Model continuous growth and decay with exponential functions.

HIGH SCHOOL · MA.912.C.5.3

MA.912.C.5.3 "Solve differential equations of the form dY/dt=kY as applied to growth and decay problems."

NAME _____ DATE _____

THE BIG IDEA

When dY/dt=kY, the quantity changes by a constant fraction of its current amount. If $Y(0)=Y_0$, the solution is $Y=Y_0e^{(kt)}$, where positive k means growth and negative k means decay.

WORKED EXAMPLE

Solve dY/dt=13Y, given Y(0)=17.

1. Separate variables: dY/Y=13 dt.
2. Integrate: $\ln|Y|=13t+C$, so $Y=Ce^{(13t)}$.
3. Use Y(0)=17 to get C=17.

Answer: $Y=17e^{(13t)}$

PRACTICE 6 problems

1 Solve dY/dt=2Y, given Y(0)=5.

ANSWER _____

2 Solve dY/dt=-3Y, given Y(0)=8.

ANSWER _____

3 If dY/dt=(1/4)Y and Y(0)=12, find Y when t=4 ln 2.

ANSWER _____

4 If dY/dt=-(1/2)Y and Y(0)=18, find Y when t=2 ln 3.

ANSWER _____

5 Solve dY/dt=Y/3, given Y(1)=12.

ANSWER _____

6 If dY/dt=-4Y and Y(0)=1/3, find Y when t=(ln 3)/4.

ANSWER _____

APPLY IT Show your work

1 A student's music channel has 120 followers at t=0 days. Its follower count F follows dF/dt=(1/5)F. How many followers will the channel have after 5 ln 2 days?

WORK SPACE

ANSWER _____

2 A phone battery starts at 90 percent charge. Its charge B, measured in percentage points, follows dB/dt=-(1/6)B, where t is in hours. What will the charge be after 6 ln 3 hours?

WORK SPACE

ANSWER _____

CHALLENGE One step up

A streaming creator has 16 followers at t=0 days and 128 followers at t=3 days. Assume the follower count V follows dV/dt=kV. Find k, write a formula for V, and find V(4).

WORK SPACE AND ANSWER

Change Over Time

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Solve dY/dt=2Y, given Y(0)=5.	**$Y=5e^{(2t)}$**
2	Solve dY/dt=-3Y, given Y(0)=8.	**$Y=8e^{(-3t)}$**
3	If dY/dt=(1/4)Y and Y(0)=12, find Y when t=4 ln 2.	**24**
4	If dY/dt=-(1/2)Y and Y(0)=18, find Y when t=2 ln 3.	**6**
5	Solve dY/dt=Y/3, given Y(1)=12.	**$Y=12e^{((t-1)/3)}$**
6	If dY/dt=-4Y and Y(0)=1/3, find Y when t=(ln 3)/4.	**1/9**

PART 2 · APPLY IT

1. **240 followers** Use $F=120e^{(t/5)}$. At t=5 ln 2, $F=120e^{(\ln 2)}=240$.
2. **30 percent** Use $B=90e^{(-t/6)}$. At t=6 ln 3, $B=90e^{(-\ln 3)}=30$.

CHALLENGE

$k=\ln 2$; $V=16e^{((\ln 2)t)}=16(2^t)$; $V(4)=256$

Accept equivalent exponential forms, including forms with $e^{(\ln 2)}$ simplified to 2. For the challenge, accept k=(ln 8)/3 because it is equal to ln 2.