LESSON MAKER PRO

Follow the Slopes

Use slope fields to trace particular solutions.

HIGH SCHOOL · MA.912.C.5.4

MA.912.C.5.4 "Display a graphic representation of the solution to a differential equation by using slope fields, and locate particular solutions to the equation."

NAME _____ DATE _____

READ AND TRACE

A slope field uses a short segment at each point, and the segment's slope comes from dy/dx. To locate a particular solution, start at the given point and follow the nearby segments. When you can integrate the equation, use the initial point to find the constant and check the curve.

WORKED EXAMPLE

On a slope field for dy/dx=8x, locate the particular solution through (9,11).

1. Integrate: $y=4x^2+C$.
2. Substitute the point: $11=4(9)^2+C$.
3. Solve for the constant: $C=-313$.
4. Trace the curve $y=4x^2-313$ through (9,11).

Answer: $y=4x^2-313$

PRACTICE 6 problems

1 For the slope field dy/dx=x-y, find the slope of the segment at (3,-2).

ANSWER _____

2 For the slope field dy/dx=y-5, identify the horizontal line where all slope segments are flat.

ANSWER _____

3 For dy/dx=x/3, draw slope segments at the nine points where x=-6, 0, or 6 and y=-3, 0, or 3. State the slope in each vertical column.

ANSWER _____

4 Sketch a slope field for dy/dx=6x. Then find the particular solution that passes through (5,7).

ANSWER _____

5 Sketch several slope segments for dy/dx=y. Locate the particular solution through (0,3).

ANSWER _____

6 For dy/dx=2x/y, draw slope segments at (0,6), (6,6), (0,-6), and (6,-6). Then locate the particular solution through (0,6).

ANSWER _____

APPLY IT Show your work

1 Your phone is downloading a video. After t minutes, the downloaded amount D, in megabytes, changes at the rate dD/dt=6t. At t=0, D=12. Sketch a slope field in the t-D plane, trace the particular solution, and find D after 3 minutes.

WORK SPACE

ANSWER _____

2 You are saving for concert tickets. After t weeks, your savings S, in dollars, changes at the rate dS/dt=5t+3. At t=0, S=10. Sketch a slope field in the t-S plane, trace the particular solution, and find S after 2 weeks.

WORK SPACE

ANSWER _____

CHALLENGE One step up

A slope field for dy/dx=x/y has no segments on y=0. Find the particular solution through (0,3), and explain why you choose one branch of the relation.

WORK SPACE AND ANSWER

ANSWER KEY

Follow the Slopes

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For the slope field dy/dx=x-y, find the slope of the segment at (3,-2).	**5**
2	For the slope field dy/dx=y-5, identify the horizontal line where all slope segments are flat.	**y=5**
3	For dy/dx=x/3, draw slope segments at the nine points where x=-6, 0, or 6 and y=-3, 0, or 3. State the slope in each vertical column.	**At x=-6, every slope is -2. At x=0, every slope is 0. At x=6, every slope is 2.**
4	Sketch a slope field for dy/dx=6x. Then find the particular solution that passes through (5,7).	**$y=3x^2-68$**
5	Sketch several slope segments for dy/dx=y. Locate the particular solution through (0,3).	**$y=3e^x$**
6	For dy/dx=2x/y, draw slope segments at (0,6), (6,6), (0,-6), and (6,-6). Then locate the particular solution through (0,6).	**The slopes are 0, 2, 0, and -2, respectively. The particular solution is y=sqrt(2x²+36).**

PART 2 · APPLY IT

1. **$D(t)=3t^2+12$, so D(3)=39 megabytes.** Integrate dD/dt=6t to get $D=3t^2+C$. Use D(0)=12, then evaluate the particular solution at t=3.
2. **$S(t)=(5/2)t^2+3t+10$, so S(2)=26 dollars.** Integrate dS/dt=5t+3 to get $S=(5/2)t^2+3t+C$. Use S(0)=10, then evaluate the particular solution at t=2.

CHALLENGE

$y=\sqrt{x^2+9}$. Separating gives $y^2=x^2+C$, and y(0)=3 selects C=9 and the positive branch, which never reaches y=0.

For sketch prompts, accept slope segments with the stated slopes and a smooth curve through the given initial point. Accept algebraically equivalent solution forms, but require the positive branch when the initial value has positive y.