

Area Under Control

Use definite integrals to measure regions.

HIGH SCHOOL • MA.912.C.5.5

MA.912.C.5.5 "Find the area between a curve and the x-axis or between two curves by using definite integrals."

NAME _____

DATE _____

MEASURE THE REGION

To find area between a curve and the x-axis, integrate the function if it stays above the axis. If it crosses the x-axis, split the integral at each crossing and add the positive areas. Between two curves, integrate top function minus bottom function.

WORKED EXAMPLE

Find the area between $y = 5x + 8$ and the x-axis from $x = 2$ to $x = 6$.

1. $A = \text{integral from 2 to 6 of } (5x + 8) \, dx.$

2. $A = [(5/2)x^2 + 8x] \text{ from 2 to 6.}$

3. $A = (90 + 48) - (10 + 16) = 112.$

Answer: 112 square units

PRACTICE 6 problems

- 1** Find the area between $y = x^2$ and the x-axis from $x = 0$ to $x = 3$.

ANSWER _____

- 2** Find the area between $y = 4x + 1$ and the x-axis from $x = 0$ to $x = 3$.

ANSWER _____

- 3** Find the area between $y = 7 - x$ and the x-axis from $x = 0$ to $x = 7$.

ANSWER _____

- 4** Find the area between $y = 9$ and $y = x^2$ from $x = -3$ to $x = 3$.

ANSWER _____

- 5** Find the area between $y = 3x$ and $y = x^2$ from $x = 0$ to $x = 3$.

ANSWER _____

- 6** Find the total area between $y = x - 4$ and the x-axis from $x = -3$ to $x = 7$.

ANSWER _____

APPLY IT Show your work

- 1** A skateboarding app models a rider's velocity by $v(t) = t - 3$, where t is time in seconds and v is meters per second. Find the total distance the rider travels from $t = 0$ to $t = 7$.

WORK SPACE

ANSWER _____

- 2** A student plans a painted region for a school mural. Its top edge is $y = 12 - x^2$ and its bottom edge is $y = 3x$, with x measured in feet from $x = 0$ to $x = 3$. Find the area of the painted region.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Find the total area between $y = x^2 - 4x + 3$ and the x -axis from $x = 0$ to $x = 5$. Show where you split the integral.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Find the area between $y = x^2$ and the x-axis from $x = 0$ to $x = 3$.	9 square units
2	Find the area between $y = 4x + 1$ and the x-axis from $x = 0$ to $x = 3$.	21 square units
3	Find the area between $y = 7 - x$ and the x-axis from $x = 0$ to $x = 7$.	49/2 square units
4	Find the area between $y = 9$ and $y = x^2$ from $x = -3$ to $x = 3$.	36 square units
5	Find the area between $y = 3x$ and $y = x^2$ from $x = 0$ to $x = 3$.	9/2 square units
6	Find the total area between $y = x - 4$ and the x-axis from $x = -3$ to $x = 7$.	29 square units

PART 2 • APPLY IT

- 25/2 meters** The velocity changes sign at $t = 3$, so split the area there. Add the areas $9/2$ and 8 .
- 27/2 square feet** The top curve is $12 - x^2$ and the bottom curve is $3x$ on this interval. Evaluate the integral from 0 to 3 of $(12 - x^2 - 3x) dx$.

CHALLENGE

28/3 square units. Split at $x = 1$ and $x = 3$.

Accept equivalent exact forms, such as $4 \frac{1}{2}$ for $9/2$. For regions that cross the x-axis, students must add absolute areas rather than use only the signed integral.