

Integral Averages

Use area and interval length to find a function's average value.

HIGH SCHOOL • MA.912.C.5.6

MA.912.C.5.6 "Find the average value of a function over a closed interval by using definite integrals."

NAME _____

DATE _____

AVERAGE FROM AREA

The average value of $f(x)$ on $[a,b]$ is $(1/(b-a))$ integral from a to b of $f(x)$ dx . First find the definite integral, then divide by the length of the interval, $b-a$.

WORKED EXAMPLE

Find the average value of $f(x) = 11x - 13$ on $[4,10]$.

1. Average value = $(1/(10-4))$ integral from 4 to 10 of $(11x - 13)$ dx .
2. = $1/6 [11x^2/2 - 13x]$ from 4 to 10.
3. = $1/6 ((420) - (36))$.
4. = $384/6 = 64$.

Answer: 64

PRACTICE 6 problems

- 1** Find the average value of $f(x) = x^2$ on $[-2,2]$.

ANSWER _____

- 2** Find the average value of $f(x) = 3x - 1$ on $[0,3]$.

ANSWER _____

- 3** Find the average value of $f(x) = 5 - x$ on $[1,6]$.

ANSWER _____

- 4** Find the average value of $f(x) = x^3$ on $[-1,2]$.

ANSWER _____

- 5** Find the average value of $f(x) = 2x^2 + 3$ on $[-3,0]$.

ANSWER _____

- 6** Find the average value of $f(x) = x^2 - 2x$ on $[0,5]$.

ANSWER _____

APPLY IT Show your work

- 1** A mobile game update downloads at a speed of $d(t) = 4t + 1$ megabits per second, where t is the number of seconds after the download begins. Find the average download speed from $t = 0$ to $t = 3$.

WORK SPACE

ANSWER _____

- 2** A phone charges at a rate of $r(t) = 18 - 2t$ percentage points per hour, where t is hours after it is plugged in. Find the average charging rate from $t = 1$ to $t = 7$.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A function is defined by $f(x) = x + 2$ for $0 \leq x \leq 2$, and $f(x) = 6 - x$ for $2 < x \leq 5$. Find the average value of $f(x)$ on $[0, 5]$. Show how you split the integral.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Find the average value of $f(x) = x^2$ on $[-2, 2]$.	$\frac{4}{3}$
2	Find the average value of $f(x) = 3x - 1$ on $[0, 3]$.	$\frac{7}{2}$
3	Find the average value of $f(x) = 5 - x$ on $[1, 6]$.	$\frac{3}{2}$
4	Find the average value of $f(x) = x^3$ on $[-1, 2]$.	$\frac{5}{4}$
5	Find the average value of $f(x) = 2x^2 + 3$ on $[-3, 0]$.	9
6	Find the average value of $f(x) = x^2 - 2x$ on $[0, 5]$.	$\frac{10}{3}$

PART 2 • APPLY IT

- 7 megabits per second** Compute $(1/3)$ integral from 0 to 3 of $(4t + 1) dt$. The definite integral is 21, and $21/3 = 7$.
- 10 percentage points per hour** Compute $(1/(7-1))$ integral from 1 to 7 of $(18 - 2t) dt$. The definite integral is 60, and $60/6 = 10$.

CHALLENGE

27/10

Accept equivalent exact forms, such as mixed numbers or simplified fractions. For the challenge, students should add the two definite integrals before dividing by the full interval length of 5.