LESSON MAKER PRO

Integral Averages

Use area and interval length to find a function's average value.

HIGH SCHOOL · MA.912.C.5.6

MA.912.C.5.6 "Find the average value of a function over a closed interval by using definite integrals."

NAME _____ DATE _____

AVERAGE FROM AREA

The average value of f(x) on [a,b] is (1/(b-a)) integral from a to b of f(x) dx. First find the definite integral, then divide by the length of the interval, b-a.

WORKED EXAMPLE

Find the average value of f(x) = 11x - 13 on [4,10].

1. Average value = (1/(10-4)) integral from 4 to 10 of (11x - 13) dx.
2. = 1/6 [$11x^2/2$ - 13x] from 4 to 10.
3. = 1/6 ((420) - (36)).
4. = 384/6 = 64.

Answer: 64

PRACTICE 6 problems

1 Find the average value of f(x) = x^2 on [-2,2].

ANSWER _____

2 Find the average value of f(x) = 3x - 1 on [0,3].

ANSWER _____

3 Find the average value of f(x) = 5 - x on [1,6].

ANSWER _____

4 Find the average value of f(x) = x^3 on [-1,2].

ANSWER _____

5 Find the average value of f(x) = $2x^2$ + 3 on [-3,0].

ANSWER _____

6 Find the average value of f(x) = x^2 - 2x on [0,5].

ANSWER _____

APPLY IT Show your work

1 A mobile game update downloads at a speed of d(t) = 4t + 1 megabits per second, where t is the number of seconds after the download begins. Find the average download speed from t = 0 to t = 3.

WORK SPACE

ANSWER _____

2 A phone charges at a rate of r(t) = 18 - 2t percentage points per hour, where t is hours after it is plugged in. Find the average charging rate from t = 1 to t = 7.

WORK SPACE

ANSWER _____

CHALLENGE One step up

A function is defined by f(x) = x + 2 for 0 <= x <= 2, and f(x) = 6 - x for 2 < x <= 5. Find the average value of f(x) on [0,5]. Show how you split the integral.

WORK SPACE AND ANSWER

Integral Averages

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Find the average value of $f(x) = x^2$ on [-2,2].	**4/3**
2	Find the average value of $f(x) = 3x - 1$ on [0,3].	**7/2**
3	Find the average value of $f(x) = 5 - x$ on [1,6].	**3/2**
4	Find the average value of $f(x) = x^3$ on [-1,2].	**5/4**
5	Find the average value of $f(x) = 2x^2 + 3$ on [-3,0].	**9**
6	Find the average value of $f(x) = x^2 - 2x$ on [0,5].	**10/3**

PART 2 · APPLY IT

1. **7 megabits per second** Compute (1/3) integral from 0 to 3 of (4t + 1) dt. The definite integral is 21, and 21/3 = 7.
2. **10 percentage points per hour** Compute (1/(7-1)) integral from 1 to 7 of (18 - 2t) dt. The definite integral is 60, and 60/6 = 10.

CHALLENGE

27/10

Accept equivalent exact forms, such as mixed numbers or simplified fractions. For the challenge, students should add the two definite integrals before dividing by the full interval length of 5.