

Slice, Spin, Measure

Use definite integrals to build volume from cross sections.

HIGH SCHOOL • MA.912.C.5.7

MA.912.C.5.7 "Find the volume of a figure with known cross-sectional area, including figures of revolution, by using definite integrals."

NAME _____

DATE _____

ADD THE SLICES

If a solid has cross-sectional area $A(x)$, its volume from $x=a$ to $x=b$ is the integral from a to b of $A(x) dx$. For a solid made by revolving a region around the x -axis, find the circular cross-sectional area first, often $A(x)=\pi[r(x)]^2$, then integrate.

WORKED EXAMPLE

A solid has cross-sectional area $A(x)=8x+13$ square units for x from 6 to 9. Find its volume.

1. $V = \text{integral from 6 to 9 of } (8x + 13) dx.$
2. An antiderivative of $8x + 13$ is $4x^2 + 13x.$
3. $V = [4x^2 + 13x] \text{ from 6 to 9} = 219.$

Answer: 219 cubic units

PRACTICE 6 problems

- 1** A solid has cross-sectional area $A(x)=x+1$ square centimeters for x from 0 to 3. Find its volume.

ANSWER _____

- 2** A solid has cross-sectional area $A(x)=2x$ square centimeters for x from 1 to 5. Find its volume.

ANSWER _____

- 3** A solid has cross-sectional area $A(x)=x^2$ square units for x from 1 to 3. Find its volume.

ANSWER _____

- 4** A solid has cross-sectional area $A(x)=11-x$ square centimeters for x from 2 to 5. Find its volume.

ANSWER _____

- 5** The region under $y=x+1$ from $x=0$ to $x=2$ is revolved around the x -axis. Find the volume of the solid.

ANSWER _____

- 6** The region under $y=\sqrt{x+2}$ from $x=0$ to $x=4$ is revolved around the x -axis. Find the volume of the solid.

ANSWER _____

APPLY IT Show your work

1

A student is designing a 3D-printed stand for a gaming headset. At each position x centimeters from the front, the stand has cross-sectional area $A(x)=x^2+3$ square centimeters, for x from 0 to 2. Find the volume of plastic needed.

WORK SPACE

ANSWER _____

2

The art club makes a trophy by revolving the region under $y=\sqrt{x+1}$, from $x=0$ to $x=3$, around the x -axis. Find the volume of the trophy.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

The region bounded by $y=x+2$, $y=1$, $x=0$, and $x=3$ is revolved around the x -axis. Find the volume of the solid. Use washers, since the solid has a hole through its center.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	A solid has cross-sectional area $A(x)=x+1$ square centimeters for x from 0 to 3. Find its volume.	$15/2$ cubic centimeters
2	A solid has cross-sectional area $A(x)=2x$ square centimeters for x from 1 to 5. Find its volume.	24 cubic centimeters
3	A solid has cross-sectional area $A(x)=x^2$ square units for x from 1 to 3. Find its volume.	$26/3$ cubic units
4	A solid has cross-sectional area $A(x)=11-x$ square centimeters for x from 2 to 5. Find its volume.	$45/2$ cubic centimeters
5	The region under $y=x+1$ from $x=0$ to $x=2$ is revolved around the x -axis. Find the volume of the solid.	$26\pi/3$ cubic units
6	The region under $y=\sqrt{x+2}$ from $x=0$ to $x=4$ is revolved around the x -axis. Find the volume of the solid.	16π cubic units

PART 2 • APPLY IT

- $26/3$ cubic centimeters** Integrate x^2+3 from 0 to 2. The antiderivative is $x^3/3+3x$, which gives $26/3$.
- $15\pi/2$ cubic units** Revolution creates circular cross sections with area $\pi(x+1)$. Integrate $\pi(x+1)$ from 0 to 3 to get $15\pi/2$.

CHALLENGE

 36π cubic units

Accept equivalent exact forms, such as $\pi(26/3)$ for $26\pi/3$. Students should leave answers involving π in exact form and include cubic units when units are given.