LESSON MAKER PRO

# Slice, Add, Volume

*Use definite integrals to build volume from cross sections.*

**HIGH SCHOOL · MA.912.C.5.7**

**MA.912.C.5.7** "Find the volume of a figure with known cross-sectional area, including figures of revolution, by using definite integrals."

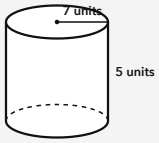
**NAME** _____ **DATE** _____

## ADD THE SLICES

If A(x) is the area of a cross section at position x, the volume is $V = \int A(x)\,dx$ over the solid's length. For a solid of revolution, first write the cross-sectional area, often pi times a radius squared.

## WORKED EXAMPLE

The cylinder shown has circular cross sections perpendicular to its height. Find its volume using a definite integral.




1. $A(x) = \pi(7)^2 = 49\pi$.
2. $V = \int_0^5 49\pi\,dx$.
3. $V = 49\pi(5)$.

**Answer: 245pi cubic units**

## PRACTICE 4 problems

**1** A solid has cross-sectional area $A(x) = x + 1$ for $0 \le x \le 3$. Find its volume using a definite integral.

**ANSWER** _____

**2** A solid has cross-sectional area $A(x) = 9 - x^2$ for $-3 \le x \le 3$. Find its volume.

**ANSWER** _____

**3** A solid has square cross sections perpendicular to the x-axis. Each square has side length x, where $0 \le x \le 4$. Find the volume.

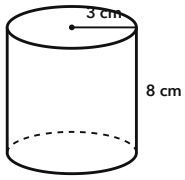
**ANSWER** _____

**4** Rotate $y = \sqrt{(x + 1)}$ about the x-axis from $x = 0$ to $x = 4$. Find the volume of the solid.

**ANSWER** _____

## APPLY IT Show your work

**1** A reusable drink bottle is modeled by the cylinder shown. Find its capacity with a definite integral.




**ANSWER** _____

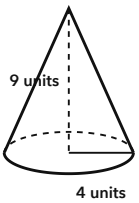
**2** A maker club designs a 3-D printed phone stand that is 3 cm long. At distance x cm from one end, its cross-sectional area is $A(x) = x^2 + 1$ square cm. Find the stand's volume.

WORK SPACE

**ANSWER** _____

## CHALLENGE One step up

For the cone shown, let x measure distance from the tip along its height. Explain why the radius is $r(x) = 4x/9$, then find the volume using a definite integral.




WORK SPACE AND ANSWER

**ANSWER KEY**

# Slice, Add, Volume

**HIGH SCHOOL · MA.912.C.5.7**

## PART 1 · PRACTICE

| # | PROBLEM | ANSWER |
|---|---|---|
| 1 | A solid has cross-sectional area $A(x) = x + 1$ for $0 \le x \le 3$. Find its volume using a definite integral. | **15/2 cubic units** |
| 2 | A solid has cross-sectional area $A(x) = 9 - x^2$ for $-3 \le x \le 3$. Find its volume. | **36 cubic units** |
| 3 | A solid has square cross sections perpendicular to the x-axis. Each square has side length x, where $0 \le x \le 4$. Find the volume. | **64/3 cubic units** |
| 4 | Rotate $y = \sqrt{(x + 1)}$ about the x-axis from $x = 0$ to $x = 4$. Find the volume of the solid. | **12pi cubic units** |

## PART 2 · APPLY IT

1. **72pi cm³** Each cross section has area $9\pi$ cm$^2$. Evaluate $\int_0^8 9\pi \, dx$.
2. **12 cm³** Evaluate $\int_0^3 (x^2 + 1) \, dx = [x^3/3 + x]_0^3$.

## CHALLENGE

**The radius grows proportionally from 0 to 4 as x goes from 0 to 9, so $r(x) = 4x/9$. $V = \int_0^9 \pi(4x/9)^2 \, dx = 48\pi$ cubic units.**

Accept equivalent exact forms and correctly evaluated definite integrals. For the challenge, accept a similar-triangles or proportional-reasoning explanation for $r(x) = 4x/9$.