LESSON MAKER PRO

Accumulation Action

Use definite integrals to measure total change, area, and distance.

HIGH SCHOOL · MA.912.C.5

MA.912.C.5 "Apply integrals to solve problems."

NAME _____ DATE _____

ACCUMULATING CHANGE

A definite integral adds a rate or quantity over an interval. If $F'(x) = f(x)$, then $\int_a^b f(x)\,dx = F(b) - F(a)$. Check the context to decide whether the result represents net change, total amount, area, or distance.

WORKED EXAMPLE

Evaluate $\int_{14}^{15} (x + 7)\,dx$.

1. An antiderivative of $x + 7$ is $x^2/2 + 7x$.
2. Substitute the upper bound: $15^2/2 + 7(15) = 435/2$.
3. Substitute the lower bound: $14^2/2 + 7(14) = 392/2$.
4. Subtract: $435/2 - 392/2 = 43/2$.

Answer: 43/2

PRACTICE 6 problems

1 Evaluate $\int_0^4 (x + 5)\,dx$.

ANSWER _____

2 Evaluate $\int_1^3 4x\,dx$.

ANSWER _____

3 Evaluate $\int_{-2}^2 (x^3 + 6)\,dx$.

ANSWER _____

4 Evaluate $\int_2^5 (3x^2 - 1)\,dx$.

ANSWER _____

5 A drone's vertical velocity is $v(t) = t + 4$ meters per second for $0 <= t <= 3$. Find its change in height.

ANSWER _____

6 Find the area between $y = 2x$ and the x-axis from $x = 0$ to $x = 4$.

ANSWER _____

APPLY IT Show your work

1 A gaming club's live-chat message rate is r(t) = 3t + 2 messages per minute, where t is the number of minutes after a tournament begins. From t = 0 to t = 4, how many messages are posted?

WORK SPACE

ANSWER _____

2 A student-designed electric skateboard moves along a straight test track with velocity $v(t) = t^2 + 2t$ meters per second for 0 <= t <= 3. How far does it move?

WORK SPACE

ANSWER _____

CHALLENGE One step up

A skateboard's velocity is v(t) = t - 2 meters per second for 0 <= t <= 5. Find its total distance traveled, not its displacement. Show how you split the integral.

WORK SPACE AND ANSWER

Accumulation Action

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Evaluate $\int_0^4 (x + 5)\,dx$.	**28**
2	Evaluate $\int_1^3 4x\,dx$.	**16**
3	Evaluate $\int_{-2}^2 (x^3 + 6)\,dx$.	**24**
4	Evaluate $\int_2^5 (3x^2 - 1)\,dx$.	**114**
5	A drone's vertical velocity is v(t) = t + 4 meters per second for 0 <= t <= 3. Find its change in height.	**33/2 meters**
6	Find the area between y = 2x and the x-axis from x = 0 to x = 4.	**16 square units**

PART 2 · APPLY IT

1. **32 messages** Integrate the message rate from 0 to 4: $[3t^2/2 + 2t]_0^4 = 32$. The integral of a rate gives the total number of messages.
2. **18 meters** The velocity is nonnegative on this interval, so distance is $\int_0^3 (t^2 + 2t)\,dt$. Evaluating $[t^3/3 + t^2]_0^3$ gives 18.

CHALLENGE

13/2 meters; split at t = 2: $\int_0^2 (2 - t)\,dt + \int_2^5 (t - 2)\,dt = 2 + 9/2 = 13/2$.

Accept equivalent exact forms, including decimal forms when correct. For the challenge, require students to split at t = 2 or otherwise account for the velocity changing from negative to positive.