

Residual Reality

Test a line by looking at its errors.

HIGH SCHOOL • MA.912.DP.2.5

MA.912.DP.2.5 "Given a scatter plot that represents bivariate numerical data, assess the fit of a given linear function by plotting and analyzing residuals."

NAME _____

DATE _____

READ THE RESIDUALS

A residual equals the observed y-value minus the y-value predicted by the model. Positive residuals are above the line, and negative residuals are below the line. A linear model fits well when residuals are scattered around 0 with no clear pattern.

WORKED EXAMPLE

For the observed point (29,341), the model is $y = 11x + 18$. Find the residual.

1. Find the predicted y-value: $11(29) + 18 = 337$.
2. Subtract predicted from observed: $341 - 337 = 4$.
3. The positive residual means the point is above the model line.

Answer: 4

PRACTICE 6 problems

- 1** For the observed point (7,25), the model is $y = 3x + 5$. Find the residual.

ANSWER _____

- 2** For the observed point (8,31), the model is $y = 3x + 6$. Find the residual.

ANSWER _____

- 3** For the observed point (9,35), the model is $y = 3x + 8$. Find the residual and state where the point is relative to the model line.

ANSWER _____

- 4** A residual plot has points (1,-3), (5,-1), (9,1), and (13,3). Describe the pattern and assess the linear model.

ANSWER _____

- 5** A residual plot has points (2,-1), (6,2), (10,-2), and (14,1). Assess the linear model.

ANSWER _____

- 6** A residual plot has points (3,5), (6,2), (9,0), (12,-2), and (15,-5). Describe the pattern and assess the linear model.

ANSWER _____

APPLY IT Show your work

- 1** A student band models y , the number of hundreds of song streams in a week, from x , the number of social media posts. Its model is $y = 2x + 7$. The observed data are (1,8), (3,12), (5,16), and (7,20). List the residual-plot points, then assess the model.

WORK SPACE

ANSWER _____

- 2** A robotics team models y , the number of successful test runs, from x , the number of extra testing sessions. Its model is $y = x + 2$. The observed data are (1,2), (2,4), (3,5), and (4,7). List the residual-plot points, then assess the model's slope.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A model is $y = 2x + 9$. Its residual plot has points (1,-3), (3,-1), (5,1), and (7,3). Explain what the pattern says about the model, then write a revised linear function that fits all four data points.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	For the observed point (7,25), the model is $y = 3x + 5$. Find the residual.	-1
2	For the observed point (8,31), the model is $y = 3x + 6$. Find the residual.	1
3	For the observed point (9,35), the model is $y = 3x + 8$. Find the residual and state where the point is relative to the model line.	0; the point is on the model line.
4	A residual plot has points (1,-3), (5,-1), (9,1), and (13,3). Describe the pattern and assess the linear model.	The residuals increase as x increases, so there is a pattern. The model is not a good fit because it overpredicts at lower x-values and underpredicts at higher x-values. Its slope is too shallow.
5	A residual plot has points (2,-1), (6,2), (10,-2), and (14,1). Assess the linear model.	The residuals are scattered above and below 0 with no clear pattern, so the linear model is a reasonable fit.
6	A residual plot has points (3,5), (6,2), (9,0), (12,-2), and (15,-5). Describe the pattern and assess the linear model.	The residuals decrease as x increases, so there is a pattern. The model is not a good fit and its slope is too steep.

PART 2 • APPLY IT

- Residual-plot points: (1,-1), (3,-1), (5,-1), (7,-1). The model consistently overpredicts by 1 hundred streams, so it is not centered around 0. A better model for these data is $y = 2x + 6$.** Find each predicted value from $y = 2x + 7$, then subtract it from the observed value. Every residual is -1, showing a consistent bias.
- Residual-plot points: (1,-1), (2,0), (3,0), (4,1). The residuals show an overall upward trend, so the model underpredicts more as x increases. The slope of $y = x + 2$ is too shallow.** The residuals are observed y minus predicted y, giving -1, 0, 0, and 1. Their overall increase shows that the model needs a greater slope.

CHALLENGE

The residuals increase by 2 when x increases by 2, so the model's slope is 1 too small. A revised function is $y = 3x + 5$. It gives a residual of 0 for all four points.

Accept equivalent descriptions of residual patterns, including that a rising pattern means the original slope is too small and a falling pattern means it is too large. Accept residual points written as ordered pairs or in a table.