LESSON MAKER PRO

Curve Fit Quest

Build quadratic models, then use their key points.

HIGH SCHOOL · MA.912.DP.2.8

MA.912.DP.2.8 "Fit a quadratic function to bivariate numerical data that suggests a quadratic association and interpret any intercepts or the vertex of the model. Use the model to solve real-world problems in terms of the context of the data."

NAME _____ DATE _____

FIT, THEN INTERPRET

Use y=ax^2+bx+c and substitute three data points to solve for a, b, and c. The vertex gives the maximum or minimum value, the y-intercept is the value when x=0, and x-intercepts are where the modeled quantity is 0.

WORKED EXAMPLE

The points (0, 5), (1, 12), and (2, 21) lie on a quadratic model y=ax^2+bx+c. Find the model, its vertex, and its intercepts.

1. At x=0, c=5.
2. At x=1, a+b+5=12, so a+b=7.
3. At x=2, 4a+2b+5=21, so 2a+b=8; therefore a=1 and b=6.
4. The model is y=x^2+6x+5=(x+3)^2-4.

Answer: y=x^2+6x+5; vertex (-3, -4); y-intercept (0, 5); x-intercepts (-5, 0) and (-1, 0).

PRACTICE 6 problems

1 The points (0, 4), (1, 1), and (2, 0) lie on y=ax^2+bx+c. Find the quadratic model.

ANSWER _____

2 The points (0, -2), (1, 1), and (2, 6) lie on y=ax^2+bx+c. Find the model and state its vertex.

ANSWER _____

3 The points (0, -8), (1, -3), and (2, 0) lie on a quadratic model. Find the model and its x-intercepts.

ANSWER _____

4 The points (0, 9), (1, 4), and (2, 1) lie on a quadratic model. Find the model, its vertex, and its y-intercept.

ANSWER _____

5 A quadratic fitted to data is y=-2x^2+8x+3. Find the vertex and y-intercept.

ANSWER _____

6 A quadratic fitted to data is y=x^2-10x+16. Find the x-intercepts and vertex.

ANSWER _____

APPLY IT Show your work

1 A science club launches a small water rocket. Its height h, in feet, is recorded at these times t, in seconds: (0, 11), (1, 20), and (2, 27). Assume the data fit h=at^2+bt+c. Find the model. What is the rocket's maximum height, and when does it return to the ground?

WORK SPACE

ANSWER _____

2 A student council models its profit P, in hundreds of dollars, based on ticket price p, in dollars. The data points are (2, 0), (3, 9), and (4, 16). Assume P=ap^2+bp+c. Find the model, the ticket prices that break even, and the ticket price and profit that maximize profit.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A music venue models profit P, in hundreds of dollars, by ticket price x, in dollars. The data points are (1, 0), (3, 8), and (5, 8). Fit a quadratic model. The venue will consider only whole-dollar ticket prices from $2 through $6. What price gives the greatest profit, and explain why neither break-even price is an option?

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_____

ANSWER KEY

Curve Fit Quest

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	The points (0, 4), (1, 1), and (2, 0) lie on y=ax^2+bx+c. Find the quadratic model.	**y=x^2-4x+4**
2	The points (0, -2), (1, 1), and (2, 6) lie on y=ax^2+bx+c. Find the model and state its vertex.	**y=x^2+2x-2; vertex (-1, -3)**
3	The points (0, -8), (1, -3), and (2, 0) lie on a quadratic model. Find the model and its x-intercepts.	**y=-x^2+6x-8; x-intercepts (2, 0) and (4, 0)**
4	The points (0, 9), (1, 4), and (2, 1) lie on a quadratic model. Find the model, its vertex, and its y-intercept.	**y=x^2-6x+9; vertex (3, 0); y-intercept (0, 9)**
5	A quadratic fitted to data is y=-2x^2+8x+3. Find the vertex and y-intercept.	**vertex (2, 11); y-intercept (0, 3)**
6	A quadratic fitted to data is y=x^2-10x+16. Find the x-intercepts and vertex.	**x-intercepts (2, 0) and (8, 0); vertex (5, -9)**

PART 2 · APPLY IT

1. **h=-t^2+10t+11; maximum height 36 feet at 5 seconds; the rocket returns to the ground at 11 seconds.** Substituting the three points gives a=-1, b=10, and c=11. The vertex is (5, 36), and h=0 has roots t=-1 and t=11, so only 11 seconds is meaningful.
2. **P=-p^2+14p-24; break-even prices are $2 and $12; maximum profit is $2,500 at a ticket price of $7.** Substitution gives P=-p^2+14p-24. Its x-intercepts are p=2 and p=12, and its vertex is (7, 25), where 25 represents $2,500.

CHALLENGE

P=-x^2+8x-7. The greatest profit occurs at x=4, where P=9, or $900. The break-even prices are $1 and $7, but both are outside the allowed price range of $2 through $6.

Accept equivalent quadratic forms, including factored and vertex form, when they represent the same model. For contextual questions, accept only nonnegative time values and use the stated units when interpreting coordinates.