LESSON MAKER PRO

Quadratic Data Detectives

Fit a parabola, then use its key points.

HIGH SCHOOL · MA.912.DP.2.8

MA.912.DP.2.8 "Fit a quadratic function to bivariate numerical data that suggests a quadratic association and interpret any intercepts or the vertex of the model. Use the model to solve real-world problems in terms of the context of the data."

NAME _____ DATE _____

SPOT THE PATTERN

When x-values are equally spaced, quadratic data have constant second differences. Fit $y = ax^2 + bx + c$, then use x-intercepts when $y = 0$ and the vertex for a maximum or minimum.

WORKED EXAMPLE

Fit a quadratic model to the points (6, 73), (7, 90), and (8, 109).

1. The first differences are 17 and 19, so the second difference is 2.
2. Since x increases by 1, $2a = 2$, so $a = 1$.
3. Subtracting $36 + 6b + c = 73$ from $49 + 7b + c = 90$ gives $b = 4$.
4. Use $36 + 24 + c = 73$, so $c = 13$.

Answer: $y = x^2 + 4x + 13$

PRACTICE 4 problems

1 Fit a quadratic model to (0, 1), (1, 4), and (2, 9).

ANSWER _____

2 For $y = x^2 - 4x + 3$, find the x-intercepts and the vertex.

ANSWER _____

3 Fit a quadratic model to (0, 12), (1, 13), (2, 12), and (3, 9).

ANSWER _____

4 For $y = x^2 - 2x - 15$, find the y-intercept.

ANSWER _____

APPLY IT Show your work

1 A basketball's height is recorded after a toss. Let x be time in seconds and y be height in feet. Fit a model. What is the greatest height, and when does it occur?

BASKETBALL TOSS

x (seconds)	y (feet)
0	3
1	8
2	11
3	12
4	11

ANSWER _____

2 A student council tests ticket-price increases for a concert. Let x be the increase in dollars and y be profit in hundreds of dollars: (0, -16), (2, 0), (4, 8), (6, 8), (8, 0). Fit a model. Which increase gives the greatest profit?

WORK SPACE

ANSWER _____

CHALLENGE One step up

A sensor reports (0, 4), (1, 9), (2, 16), (3, 25), and (4, 38). One point is likely an error. Identify it, give the quadratic model for the other points, and state the corrected y-value.

WORK SPACE AND ANSWER

ANSWER KEY

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Fit a quadratic model to (0, 1), (1, 4), and (2, 9).	**$y = x^2 + 2x + 1$**
2	For $y = x^2 - 4x + 3$, find the x-intercepts and the vertex.	**x-intercepts: (1, 0) and (3, 0); vertex: (2, -1)**
3	Fit a quadratic model to (0, 12), (1, 13), (2, 12), and (3, 9).	**$y = -x^2 + 2x + 12$**
4	For $y = x^2 - 2x - 15$, find the y-intercept.	**(0, -15)**

PART 2 · APPLY IT

1. **$y = -x^2 + 6x + 3$; greatest height: 12 feet at 3 seconds** The constant second difference is -2, so a = -1. The model has vertex (3, 12), which gives the greatest height.
2. **$y = -x^2 + 10x - 16$; a $5 increase gives a greatest profit of $900** The data fit $y = -x^2 + 10x - 16$. Its vertex is (5, 9), and y is measured in hundreds of dollars.

CHALLENGE

(4, 38) is the error. The model is $y = x^2 + 4x + 4$, so the corrected y-value at x = 4 is 36.

Accept equivalent forms, including $y = (x + 1)^2$ for Practice 1 and $y = -(x - 2)(x - 8)$ for the ticket model. For the challenge, accept an explanation that the first four points have constant second difference 2.