

Probability Connection Check

Use conditional probability to test independence.

HIGH SCHOOL • MA.912.DP.4.4

MA.912.DP.4.4 "Interpret the independence of two events using conditional probability."

NAME _____

DATE _____

THE INDEPENDENCE TEST

To test whether events A and B are independent, compare $P(A|B)$ with $P(A)$. If they are equal, knowing that B happened does not change the chance of A, so the events are independent. You can also check whether $P(A \text{ and } B) = P(A)P(B)$.

WORKED EXAMPLE

At a school, 52 students are surveyed. Event A is "the student takes art," and 13 students take art. Event B is "the student rides the bus," and 28 students ride the bus. Of the bus riders, 7 take art. Are A and B independent?

1. Find the overall probability: $P(A) = 13/52 = 1/4$.
2. Find the conditional probability: $P(A|B) = 7/28 = 1/4$.
3. Since $P(A|B) = P(A)$, the events are independent.

Answer: Yes, A and B are independent.

PRACTICE 6 problems

- 1** Events R and S have $P(R) = 5/9$ and $P(R|S) = 5/9$. Are R and S independent?

ANSWER _____

- 2** Events M and N have $P(M) = 3/10$ and $P(M|N) = 6/10$. Are M and N independent?

ANSWER _____

- 3** In a group of 40 students, 24 are in event A, 20 are in event B, and 12 are in both events. Are A and B independent?

ANSWER _____

- 4** Given $P(X) = 5/12$, $P(Y) = 2/3$, and $P(X \text{ and } Y) = 5/18$, are X and Y independent?

ANSWER _____

- 5** If $P(J) = 8/17$ and events J and K are independent, find $P(J|K)$.

ANSWER _____

- 6** A random song is marked explicit, E, with $P(E) = 9/16$. Among songs by artist G, $P(E|G) = 3/8$. Are E and G independent?

ANSWER _____

APPLY IT Show your work

- 1** An online music club has 60 members. Of the members, 25 create playlists, 24 post concert reviews, and 10 do both. Are the events “creates a playlist” and “posts a concert review” independent? Show the probabilities you compare.

WORK SPACE

ANSWER _____

- 2** At a media club, 90 students are surveyed. Of them, 36 share short videos, 45 use a photo-editing app, and 15 do both. Are the events “shares short videos” and “uses a photo-editing app” independent? Show the probabilities you compare.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

At esports tryouts, 80 students participate. Of them, 30 choose a strategy workshop, A, and 32 choose a team-communication workshop, B. Assume A and B are independent. How many students chose both workshops, and what is $P(A|B)$? Explain your reasoning.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Events R and S have $P(R) = 5/9$ and $P(R S) = 5/9$. Are R and S independent?	Yes, because $P(R S) = P(R)$.
2	Events M and N have $P(M) = 3/10$ and $P(M N) = 6/10$. Are M and N independent?	No, because $P(M N)$ does not equal $P(M)$.
3	In a group of 40 students, 24 are in event A, 20 are in event B, and 12 are in both events. Are A and B independent?	Yes. $P(A) = 24/40 = 3/5$ and $P(A B) = 12/20 = 3/5$.
4	Given $P(X) = 5/12$, $P(Y) = 2/3$, and $P(X \text{ and } Y) = 5/18$, are X and Y independent?	Yes, because $P(X)P(Y) = (5/12)(2/3) = 5/18$.
5	If $P(J) = 8/17$ and events J and K are independent, find $P(J K)$.	8/17
6	A random song is marked explicit, E, with $P(E) = 9/16$. Among songs by artist G, $P(E G) = 3/8$. Are E and G independent?	No, because $3/8$ does not equal $9/16$.

PART 2 • APPLY IT

- Yes. $P(\text{playlist}) = 25/60 = 5/12$, and $P(\text{playlist}|\text{review}) = 10/24 = 5/12$.** Compare the overall playlist probability with the playlist probability among members who post reviews. Both probabilities are $5/12$, so the events are independent.
- No. $P(\text{short videos}) = 36/90 = 2/5$, and $P(\text{short videos}|\text{photo app}) = 15/45 = 1/3$.** The conditional probability of sharing short videos among photo-app users is $1/3$. Since $1/3$ is not equal to $2/5$, the events are not independent.

CHALLENGE

12 students chose both workshops, and $P(A|B) = 3/8$ because independence makes $P(A|B)$ equal to $P(A)$.

Accept equivalent fractions and correct comparison statements. For independence, accept either $P(A|B) = P(A)$ or $P(A \text{ and } B) = P(A)P(B)$ as valid reasoning.