

Chance Value Lab

Build distributions and find their long-run means.

HIGH SCHOOL • MA.912.DP.6.2

MA.912.DP.6.2 "Develop a probability distribution for a discrete random variable using theoretical probabilities. Find the expected value and interpret it as the mean of the discrete distribution."

NAME _____

DATE _____

EXPECTED VALUE

For each possible value of X , find its theoretical probability. Multiply each value by its probability and add: $E(X) = \text{sum of } xP(x)$. The expected value is the mean of the distribution over many repetitions.

WORKED EXAMPLE

A fair spinner has 3 equal sections labeled 11, 13, and 17. Let X be the label landed on. Make the distribution and find $E(X)$.

1. Each label has probability $1/3$.
2. Distribution: $X=11, 13, 17$ and $P(X)=1/3, 1/3, 1/3$.
3. $E(X)=11(1/3)+13(1/3)+17(1/3)=41/3$.

Answer: $E(X)=41/3$. The long-run mean label is $41/3$.

PRACTICE 5 problems

- 1** Flip a fair coin twice. Let X be the number of heads. Write the distribution and find $E(X)$.

ANSWER _____

- 2** A bag has 1 red, 2 blue, and 3 green marbles. Pick one. Let $X=0$ for red, $X=2$ for blue, and $X=4$ for green. Write the distribution and find $E(X)$.

ANSWER _____

- 3** Roll a fair die. Let $X=1$ if the roll is even and $X=0$ otherwise. Write the distribution and find $E(X)$.

ANSWER _____

- 4** Flip a fair coin 3 times. Let X be the number of heads. Write the distribution and find $E(X)$.

ANSWER _____

- 5** A fair spinner has 4 equal sections labeled -1, 0, 1, and 6. Let X be the label landed on. Write the distribution and find $E(X)$.

ANSWER _____

APPLY IT Show your work

- 1** A mobile game tournament gives a player one reward code chosen from 10 equally likely codes. Five codes give 0 coins, three give 2 coins, and two give 5 coins. Let X be the number of coins. Write the distribution, find $E(X)$, and interpret it.

WORK SPACE

ANSWER _____

- 2** A school cafe has 12 sealed discount cards. Four cards give \$0 off, six give \$1 off, and two give \$4 off. Let X be the discount. Write the distribution, find $E(X)$, and interpret it.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

Two fair dice are rolled. Let $X=0$ if the sum is 2 through 5, $X=4$ if the sum is 6 through 9, and $X=10$ if the sum is 10 through 12. Develop the distribution, find $E(X)$, and explain what it means.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Flip a fair coin twice. Let X be the number of heads. Write the distribution and find $E(X)$.	$X=0, 1, 2$; $P(X)=1/4, 1/2, 1/4$; $E(X)=1$.
2	A bag has 1 red, 2 blue, and 3 green marbles. Pick one. Let $X=0$ for red, $X=2$ for blue, and $X=4$ for green. Write the distribution and find $E(X)$.	$X=0, 2, 4$; $P(X)=1/6, 1/3, 1/2$; $E(X)=8/3$.
3	Roll a fair die. Let $X=1$ if the roll is even and $X=0$ otherwise. Write the distribution and find $E(X)$.	$X=0, 1$; $P(X)=1/2, 1/2$; $E(X)=1/2$.
4	Flip a fair coin 3 times. Let X be the number of heads. Write the distribution and find $E(X)$.	$X=0, 1, 2, 3$; $P(X)=1/8, 3/8, 3/8, 1/8$; $E(X)=3/2$.
5	A fair spinner has 4 equal sections labeled -1, 0, 1, and 6. Let X be the label landed on. Write the distribution and find $E(X)$.	$X=-1, 0, 1, 6$; $P(X)=1/4$ for each value; $E(X)=3/2$.

PART 2 • APPLY IT

- $X=0, 2, 5$; $P(X)=1/2, 3/10, 1/5$; $E(X)=8/5$. Over many codes, the mean reward is $8/5$ coins per code. Use the number of each code type out of 10 for the probabilities. Compute $0(1/2)+2(3/10)+5(1/5)=8/5$.
- $X=0, 1, 4$; $P(X)=1/3, 1/2, 1/6$; $E(X)=7/6$. Over many cards, the mean discount is $\$7/6$ per card. Use the card counts out of 12 for the probabilities. Compute $0(1/3)+1(1/2)+4(1/6)=7/6$.

CHALLENGE

$X=0, 4, 10$; $P(X)=5/18, 5/9, 1/6$; $E(X)=35/9$. Over many rolls, the mean score is $35/9$ points per roll.

Accept equivalent fractions and equivalent probability forms. For interpretations, accept wording that identifies expected value as the long-run mean, not a guaranteed result of one trial.