

Data to Mean

Use trial results to build distributions and find their mean.

HIGH SCHOOL • MA.912.DP.6.3

MA.912.DP.6.3 "Develop a probability distribution for a discrete random variable using empirical probabilities. Find the expected value and interpret it as the mean of the discrete distribution."

NAME _____

DATE _____

EMPIRICAL DISTRIBUTIONS

Find each empirical probability by dividing its frequency by the total number of trials. Multiply each value by its probability and add the products to find $E(x)$, the distribution's mean.

WORKED EXAMPLE

In 12 trials, $x = 1$ occurred 2 times, $x = 3$ occurred 4 times, and $x = 5$ occurred 6 times. Find the distribution and $E(x)$.

- Total trials = $2 + 4 + 6 = 12$.
- $P(1) = 2/12 = 1/6$, $P(3) = 4/12 = 1/3$, $P(5) = 6/12 = 1/2$.
- $E(x) = 1(1/6) + 3(1/3) + 5(1/2) = 11/3$.

Answer: Distribution: $P(1)=1/6$, $P(3)=1/3$, $P(5)=1/2$. $E(x)=11/3$.

PRACTICE 5 problems

- 1** In 24 trials, $x = 0$ occurred 9 times, $x = 7$ occurred 8 times, and $x = 8$ occurred 7 times. Find the distribution and $E(x)$.

ANSWER _____

- 2** In 30 trials, $x = 0$ occurred 11 times, $x = 7$ occurred 10 times, and $x = 9$ occurred 9 times. Find the distribution and $E(x)$.

ANSWER _____

- 3** In 27 trials, $x = 0$ occurred 11 times, $x = 8$ occurred 9 times, and $x = 10$ occurred 7 times. Find the distribution and $E(x)$.

ANSWER _____

- 4** In 39 trials, $x = 0$ occurred 8 times, $x = 7$ occurred 20 times, and $x = 9$ occurred 11 times. Find the distribution and $E(x)$.

ANSWER _____

- 5** In 15 trials, $x = 0$ occurred 7 times and $x = 8$ occurred 8 times. Find the distribution and $E(x)$.

ANSWER _____

APPLY IT Show your work

- 1** During 30 weekly playlist updates, a student added 0 tracks in 11 weeks, 7 tracks in 10 weeks, and 9 tracks in 9 weeks. Build the empirical distribution and find the expected number of tracks added per update.

WORK SPACE

ANSWER _____

- 2** Over 30 games, a basketball team made 0 three-pointers in 13 games, 7 three-pointers in 9 games, and 8 three-pointers in 8 games. Build the empirical distribution and find the expected number of three-pointers per game.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

In 24 trials, $x = 10$ occurred 9 times, $x = 13$ occurred 8 times, and $x = 16$ occurred 7 times. Find the distribution and $E(x)$. A classmate says $E(x)$ must be the most likely result. Explain why the classmate is incorrect.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	In 24 trials, $x = 0$ occurred 9 times, $x = 7$ occurred 8 times, and $x = 8$ occurred 7 times. Find the distribution and $E(x)$.	$P(0)=3/8$, $P(7)=1/3$, $P(8)=7/24$; $E(x)=14/3$.
2	In 30 trials, $x = 0$ occurred 11 times, $x = 7$ occurred 10 times, and $x = 9$ occurred 9 times. Find the distribution and $E(x)$.	$P(0)=11/30$, $P(7)=1/3$, $P(9)=3/10$; $E(x)=151/30$.
3	In 27 trials, $x = 0$ occurred 11 times, $x = 8$ occurred 9 times, and $x = 10$ occurred 7 times. Find the distribution and $E(x)$.	$P(0)=11/27$, $P(8)=1/3$, $P(10)=7/27$; $E(x)=142/27$.
4	In 39 trials, $x = 0$ occurred 8 times, $x = 7$ occurred 20 times, and $x = 9$ occurred 11 times. Find the distribution and $E(x)$.	$P(0)=8/39$, $P(7)=20/39$, $P(9)=11/39$; $E(x)=239/39$.
5	In 15 trials, $x = 0$ occurred 7 times and $x = 8$ occurred 8 times. Find the distribution and $E(x)$.	$P(0)=7/15$, $P(8)=8/15$; $E(x)=64/15$.

PART 2 • APPLY IT

- $P(0)=11/30$, $P(7)=1/3$, $P(9)=3/10$; $E(x)=151/30$ tracks per update.** Divide each frequency by 30. Then calculate $0(11/30) + 7(1/3) + 9(3/10) = 151/30$.
- $P(0)=13/30$, $P(7)=3/10$, $P(8)=4/15$; $E(x)=127/30$ three-pointers per game.** Divide each frequency by 30. Then calculate $0(13/30) + 7(3/10) + 8(4/15) = 127/30$.

CHALLENGE

$P(10)=3/8$, $P(13)=1/3$, $P(16)=7/24$; $E(x)=51/4$. The most likely result is 10 because it has probability $3/8$. The expected value, $51/4$, is the mean and does not have to be an observed result.

Accept equivalent unsimplified probabilities and equivalent expected-value fractions. Accept a decimal interpretation only if it is accurate and clearly identified as an average.