

First Success Quest

Use geometric distributions to find when a first success happens.

HIGH SCHOOL • MA.912.DP.6.5

MA.912.DP.6.5 "Solve real-world problems involving geometric distributions."

NAME _____

DATE _____

THE FIRST-SUCCESS RULE

A geometric distribution counts the trial on which the first success occurs when each independent trial has the same probability p of success. To find the probability that the first success is on trial k , use $P(X = k) = (1 - p)^{(k - 1)} * p$, because the earlier trials fail and trial k succeeds.

WORKED EXAMPLE

An online practice tool marks an answer correct with probability $3/4$ on each independent attempt. What is the probability that the first correct answer is on the third attempt?

1. $P(X = 3) = (1 - 3/4)^{(3 - 1)} * 3/4$

2. $= (1/4)^2 * 3/4$

3. $= 3/64$

Answer: 3/64

PRACTICE 6 problems

- 1** Each independent trial has a probability of $1/2$ of success. Find the probability that the first success occurs on the second trial.

ANSWER _____

- 2** Each independent trial has a probability of $1/5$ of success. Find the probability that the first success occurs on the fourth trial.

ANSWER _____

- 3** Each independent trial has a probability of $2/3$ of success. Find the probability that the first success occurs on the second trial.

ANSWER _____

- 4** Each independent trial has a probability of $3/5$ of success. Find the probability that the first success occurs on the fifth trial.

ANSWER _____

- 5** Each independent trial has a probability of $1/4$ of success. Find the probability that the first success occurs on the sixth trial.

ANSWER _____

- 6** Each independent trial has a probability of $2/5$ of success. Find the probability that the first success occurs within the first two trials.

ANSWER _____

APPLY IT Show your work

- 1** A music app gives Maya a bonus track with probability $\frac{1}{2}$ each time she opens a mystery playlist. Assume each opening is independent. What is the probability that the first bonus track appears on her fifth opening?

WORK SPACE

ANSWER _____

- 2** A robotics team runs a sensor test repeatedly. Each independent test has a probability of $\frac{2}{5}$ of passing. What is the probability that the first passing test occurs on the fourth run?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A game server gives a rare item with probability $\frac{1}{2}$ after each independent match. What is the probability that the first rare item appears on an even-numbered match no later than the sixth match? Explain which outcomes you add.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Each independent trial has a probability of $\frac{1}{2}$ of success. Find the probability that the first success occurs on the second trial.	$\frac{1}{4}$
2	Each independent trial has a probability of $\frac{1}{5}$ of success. Find the probability that the first success occurs on the fourth trial.	$\frac{64}{625}$
3	Each independent trial has a probability of $\frac{2}{3}$ of success. Find the probability that the first success occurs on the second trial.	$\frac{2}{9}$
4	Each independent trial has a probability of $\frac{3}{5}$ of success. Find the probability that the first success occurs on the fifth trial.	$\frac{48}{3125}$
5	Each independent trial has a probability of $\frac{1}{4}$ of success. Find the probability that the first success occurs on the sixth trial.	$\frac{243}{4096}$
6	Each independent trial has a probability of $\frac{2}{5}$ of success. Find the probability that the first success occurs within the first two trials.	$\frac{16}{25}$

PART 2 • APPLY IT

1. **$\frac{1}{32}$** The first four openings must not give a bonus track, and the fifth opening must give one. Compute $(\frac{1}{2})^4 * \frac{1}{2} = \frac{1}{32}$.
2. **$\frac{54}{625}$** The first three tests must fail, then the fourth test must pass. Compute $(\frac{3}{5})^3 * \frac{2}{5} = \frac{54}{625}$.

CHALLENGE

$\frac{21}{64}$, because add $P(X = 2) + P(X = 4) + P(X = 6) = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} = \frac{21}{64}$.

Accept equivalent fraction forms, although simplified fractions are preferred. For the challenge, students should include only first successes on matches 2, 4, and 6.