LESSON MAKER PRO

First Success Quest

Use geometric distributions to find when a first success happens.

HIGH SCHOOL · MA.912.DP.6.5

MA.912.DP.6.5 "Solve real-world problems involving geometric distributions."

NAME _____ DATE _____

THE FIRST-SUCCESS RULE

A geometric distribution counts the trial on which the first success occurs when each independent trial has the same probability p of success. To find the probability that the first success is on trial k, use $P(X = k) = (1 - p)^{(k - 1)} * p$, because the earlier trials fail and trial k succeeds.

WORKED EXAMPLE

An online practice tool marks an answer correct with probability 3/4 on each independent attempt. What is the probability that the first correct answer is on the third attempt?

1. $P(X = 3) = (1 - 3/4)^{(3 - 1)} * 3/4$
2. $= (1/4)^2 * 3/4$
3. $= 3/64$

Answer: 3/64

PRACTICE 6 problems

1 Each independent trial has a probability of 1/2 of success. Find the probability that the first success occurs on the second trial.

ANSWER _____

2 Each independent trial has a probability of 1/5 of success. Find the probability that the first success occurs on the fourth trial.

ANSWER _____

3 Each independent trial has a probability of 2/3 of success. Find the probability that the first success occurs on the second trial.

ANSWER _____

4 Each independent trial has a probability of 3/5 of success. Find the probability that the first success occurs on the fifth trial.

ANSWER _____

5 Each independent trial has a probability of 1/4 of success. Find the probability that the first success occurs on the sixth trial.

ANSWER _____

6 Each independent trial has a probability of 2/5 of success. Find the probability that the first success occurs within the first two trials.

ANSWER _____

APPLY IT Show your work

1 A music app gives Maya a bonus track with probability 1/2 each time she opens a mystery playlist. Assume each opening is independent. What is the probability that the first bonus track appears on her fifth opening?

WORK SPACE

ANSWER _____

2 A robotics team runs a sensor test repeatedly. Each independent test has a probability of 2/5 of passing. What is the probability that the first passing test occurs on the fourth run?

WORK SPACE

ANSWER _____

CHALLENGE One step up

A game server gives a rare item with probability 1/2 after each independent match. What is the probability that the first rare item appears on an even-numbered match no later than the sixth match? Explain which outcomes you add.

WORK SPACE AND ANSWER

First Success Quest

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Each independent trial has a probability of 1/2 of success. Find the probability that the first success occurs on the second trial.	**1/4**
2	Each independent trial has a probability of 1/5 of success. Find the probability that the first success occurs on the fourth trial.	**64/625**
3	Each independent trial has a probability of 2/3 of success. Find the probability that the first success occurs on the second trial.	**2/9**
4	Each independent trial has a probability of 3/5 of success. Find the probability that the first success occurs on the fifth trial.	**48/3125**
5	Each independent trial has a probability of 1/4 of success. Find the probability that the first success occurs on the sixth trial.	**243/4096**
6	Each independent trial has a probability of 2/5 of success. Find the probability that the first success occurs within the first two trials.	**16/25**

PART 2 · APPLY IT

1. **1/32** The first four openings must not give a bonus track, and the fifth opening must give one. Compute $(1/2)^4 * 1/2 = 1/32$.
2. **54/625** The first three tests must fail, then the fourth test must pass. Compute $(3/5)^3 * 2/5 = 54/625$.

CHALLENGE

21/64, because add P(X = 2) + P(X = 4) + P(X = 6) = 1/4 + 1/16 + 1/64 = 21/64.

Accept equivalent fraction forms, although simplified fractions are preferred. For the challenge, students should include only first successes on matches 2, 4, and 6.