LESSON MAKER PRO

Counting Random Events

Use a Poisson model to find exact event probabilities.

HIGH SCHOOL · MA.912.DP.6.6

MA.912.DP.6.6 "Solve real-world problems involving Poisson distributions."

NAME _____ DATE _____

POISSON PROBABILITY

Use a Poisson distribution when you count events in a fixed time or space interval and know the average count, λ. For exactly k events, use $P(X=k)=\lambda^k e^{-\lambda}/k!$; for "at most" or "at least," add the needed exact probabilities or use a complement.

WORKED EXAMPLE

A sensor records an average of 7 alerts per hour. Assuming a Poisson model, find the probability of exactly 4 alerts in the next hour.

1. Use $\lambda=7$ and $k=4$.
2. $P(X=4)=7^4 e^{-7}/4!$
3. $7^4=2401$ and $4!=24$.
4. $P(X=4)=2401e^{-7}/24$.

Answer: $2401e^{-7}/24$

PRACTICE 6 problems

1 A game server receives an average of 2 error reports per minute. Assuming a Poisson model, find the probability of exactly 0 error reports in the next minute.

ANSWER _____

2 A bike-rental kiosk has an average of 2 rentals per hour. Assuming a Poisson model, find the probability of exactly 1 rental in the next hour.

ANSWER _____

3 A cafeteria receives an average of 3 mobile orders per minute. Assuming a Poisson model, find the probability of exactly 2 orders in the next minute.

ANSWER _____

4 A school website gets an average of 5 password-reset requests each hour. Assuming a Poisson model, find the probability of exactly 1 request in the next hour.

ANSWER _____

5 A robotics team receives an average of 1 email each hour. Assuming a Poisson model, find the probability of at most 2 emails in the next hour.

ANSWER _____

6 A transit app records an average of 6 route searches per minute. Assuming a Poisson model, find the probability of exactly 3 searches in the next minute.

ANSWER _____

APPLY IT Show your work

1 During a school esports tournament, a streaming channel gains an average of 5 new followers every 10 minutes. Assume follower gains follow a Poisson distribution. What is the probability that the channel gains exactly 3 new followers during the next 10 minutes?

WORK SPACE

ANSWER _____

2 A bike-share station near campus has an average of 2 checkouts every 5 minutes. Assume checkouts follow a Poisson distribution. What is the probability of at least 2 checkouts during the next 5 minutes?

WORK SPACE

ANSWER _____

CHALLENGE One step up

A student livestream receives an average of 3 comments per minute. Assume comments follow a Poisson distribution. Given that at least 1 comment arrives in the next minute, what is the probability that exactly 2 comments arrive?

WORK SPACE AND ANSWER

ANSWER KEY

Counting Random Events

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	A game server receives an average of 2 error reports per minute. Assuming a Poisson model, find the probability of exactly 0 error reports in the next minute.	$\mathbf{e^{-2}}$
2	A bike-rental kiosk has an average of 2 rentals per hour. Assuming a Poisson model, find the probability of exactly 1 rental in the next hour.	$\mathbf{2e^{-2}}$
3	A cafeteria receives an average of 3 mobile orders per minute. Assuming a Poisson model, find the probability of exactly 2 orders in the next minute.	$\mathbf{9e^{-3}/2}$
4	A school website gets an average of 5 password-reset requests each hour. Assuming a Poisson model, find the probability of exactly 1 request in the next hour.	$\mathbf{5e^{-5}}$
5	A robotics team receives an average of 1 email each hour. Assuming a Poisson model, find the probability of at most 2 emails in the next hour.	$\mathbf{5e^{-1}/2}$
6	A transit app records an average of 6 route searches per minute. Assuming a Poisson model, find the probability of exactly 3 searches in the next minute.	$\mathbf{36e^{-6}}$

PART 2 · APPLY IT

1. **$125e^{-5}/6$** Use $\lambda=5$ and $k=3$ in $P(X=k)=\lambda^k e^{-\lambda}/k!$. This gives $5^3 e^{-5}/3!=125e^{-5}/6$.
2. **$1-3e^{-2}$** Use the complement of 0 or 1 checkout. $P(X \text{ at least } 2)=1-[e^{-2}+2e^{-2}]=1-3e^{-2}$.

CHALLENGE

$9/(2(e^3-1))$

Accept algebraically equivalent exact forms, including unsimplified Poisson expressions. For the challenge, accept $(9e^{-3}/2)/(1-e^{-3})$ as equivalent to $9/(2(e^3-1))$.