

Risk and Reward

Use expected value and spread to compare choices.

HIGH SCHOOL • MA.912.DP.6.7

MA.912.DP.6.7 "Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values and standard deviations..."

NAME _____

DATE _____

COMPARE THE PAYOFFS

Find expected value with $E = \text{sum of (probability} \times \text{payoff)}$.
Then find standard deviation with $SD = \sqrt{\text{sum of probability} \times (\text{payoff} - E)^2}$. A higher expected value gives a higher long-run average, and a lower SD means less spread.

WORKED EXAMPLE

An event job pays \$10 with probability $\frac{1}{4}$ and \$30 with probability $\frac{3}{4}$. Find the expected payoff and standard deviation.

1. $E = (\frac{1}{4})(10) + (\frac{3}{4})(30) = 25$.
2. $\text{Variance} = (\frac{1}{4})(10 - 25)^2 + (\frac{3}{4})(30 - 25)^2$.
3. $\text{Variance} = 75$, so $SD = \sqrt{75} = 5\sqrt{3}$.

**Answer: Expected payoff = \$25;
standard deviation = $5\sqrt{3}$.**

PRACTICE 6 problems

- 1** A game has a payoff of -\$6 with probability $\frac{1}{2}$ and \$6 with probability $\frac{1}{2}$. Find E and SD.

ANSWER _____

- 2** A prize has a payoff of \$2 with probability $\frac{1}{2}$ and \$14 with probability $\frac{1}{2}$. Find E and SD.

ANSWER _____

- 3** A contest has payoffs of \$0, \$9, and \$18, each with probability $\frac{1}{3}$. Find E and SD.

ANSWER _____

- 4** A choice has a payoff of \$5 with probability $\frac{1}{2}$ and \$11 with probability $\frac{1}{2}$. Find E and SD.

ANSWER _____

- 5** Strategy A pays \$0 with probability $\frac{1}{2}$ and \$12 with probability $\frac{1}{2}$. Strategy B pays \$6 for sure. Find E and SD for each strategy. Which strategy has less variability?

ANSWER _____

- 6** A side job has payoffs of -\$4, \$8, and \$20 with probabilities $\frac{1}{6}$, $\frac{2}{3}$, and $\frac{1}{6}$. Find E and SD.

ANSWER _____

APPLY IT Show your work

- 1** Riley can accept a pet-sitting job that pays \$0 if it is canceled, with probability $\frac{1}{2}$, or \$24 if it happens, with probability $\frac{1}{2}$. Riley can instead work a scoreboard job that pays \$12 for sure. Find E and SD for both choices. Which job should Riley choose for the same expected income with less risk?

WORK SPACE

ANSWER _____

- 2** A student selling original sticker designs can use an online preorder plan or a pop-up table plan. The online plan has net profits of \$16 or \$40, each with probability $\frac{1}{2}$. The pop-up plan has net profits of \$20 or \$32, each with probability $\frac{1}{2}$. Find E and SD for both plans. Which plan has the greater expected profit, and which has less variability?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A student can enter a gaming tournament with net payoffs of -\$9, \$3, and \$33, with probabilities $\frac{1}{6}$, $\frac{1}{2}$, and $\frac{1}{3}$. The student can instead work a shift that pays \$11 for sure. Find E and SD for both choices. If earning \$11 on average is enough, which choice has lower risk? Explain using your calculations.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	A game has a payoff of -\$6 with probability 1/2 and \$6 with probability 1/2. Find E and SD.	E = \$0; SD = \$6.
2	A prize has a payoff of \$2 with probability 1/2 and \$14 with probability 1/2. Find E and SD.	E = \$8; SD = \$6.
3	A contest has payoffs of \$0, \$9, and \$18, each with probability 1/3. Find E and SD.	E = \$9; SD = $3\sqrt{6}$.
4	A choice has a payoff of \$5 with probability 1/2 and \$11 with probability 1/2. Find E and SD.	E = \$8; SD = \$3.
5	Strategy A pays \$0 with probability 1/2 and \$12 with probability 1/2. Strategy B pays \$6 for sure. Find E and SD for each strategy. Which strategy has less variability?	Strategy A: E = \$6, SD = \$6. Strategy B: E = \$6, SD = \$0. Strategy B has less variability.
6	A side job has payoffs of -\$4, \$8, and \$20 with probabilities 1/6, 2/3, and 1/6. Find E and SD.	E = \$8; SD = $4\sqrt{3}$.

PART 2 • APPLY IT

- Pet-sitting: E = \$12, SD = \$12. Scoreboard job: E = \$12, SD = \$0. Riley should choose the scoreboard job for the same expected income with less risk.** For pet-sitting, the average of \$0 and \$24 is \$12, and each payoff is \$12 from the mean. The scoreboard job is certain, so its standard deviation is 0.
- Online plan: E = \$28, SD = \$12. Pop-up plan: E = \$26, SD = \$6. The online plan has the greater expected profit, and the pop-up plan has less variability.** Each expected value is the midpoint of its two possible profits. The standard deviation is the distance from that midpoint because the two outcomes are equally likely.

CHALLENGE

Tournament: E = \$11; SD = $2\sqrt{65}$. Work shift: E = \$11; SD = \$0. Both choices have the same expected value, but the work shift has lower risk because its standard deviation is 0.

Accept equivalent exact radical forms, such as $\sqrt{48}$ for $4\sqrt{3}$. Require both expected value and standard deviation, including SD = 0 for a certain payoff.