

# Chance and Choice

*Find expected values and use them to compare outcomes.*

**HIGH SCHOOL • MA.912.DP.6**

**MA.912.DP.6** "Use probability distributions to solve problems."

NAME \_\_\_\_\_

DATE \_\_\_\_\_

## EXPECTED VALUE

A probability distribution lists each possible outcome and its probability. To find the expected value, multiply each outcome by its probability and add the products, then use that long-run average to answer the question.

## WORKED EXAMPLE

A game has score  $X$  with this distribution:  $X = 13, 17, 21$  and  $P(X) = 2/7, 3/7, 2/7$ . Find the expected score.

$$1. E(X) = 13(2/7) + 17(3/7) + 21(2/7)$$

$$2. E(X) = 26/7 + 51/7 + 42/7$$

$$3. E(X) = 119/7$$

$$4. E(X) = 17$$

**Answer: 17 points**

## PRACTICE 6 problems

- 1** A game gives  $X$  points with this distribution:  $X = 0$  or  $6$ , and  $P(X) = 3/4$  or  $1/4$ . Find  $E(X)$ .

**ANSWER** \_\_\_\_\_

- 2** A player can gain or lose points.  $X = -4, 2$ , or  $8$ , with probabilities  $2/5, 1/5$ , and  $2/5$ . Find  $E(X)$ .

**ANSWER** \_\_\_\_\_

- 3** Is this a valid probability distribution?  $X = 2, 4, 6$  and  $P(X) = 1/5, 3/10, 1/2$ . Show why.

**ANSWER** \_\_\_\_\_

- 4** A distribution has  $X = 0, 5, 10$  and  $P(X) = 1/8, p, 3/8$ . Find  $p$ , then find  $E(X)$ .

**ANSWER** \_\_\_\_\_

- 5** For  $X = 2, 4, 6, 8$ , the probabilities are  $3/10, 1/5, 3/10, 1/5$ . Find  $P(X \text{ is at least } 4)$ .

**ANSWER** \_\_\_\_\_

- 6** A practice challenge awards  $X$  points.  $X = 2, 5, 9$  and  $P(X) = 1/3, 1/2, 1/6$ . Find the expected number of points.

**ANSWER** \_\_\_\_\_

## APPLY IT Show your work

- 1** At an esports club raffle, a ticket has a  $\frac{1}{2}$  chance of winning \$0, a  $\frac{3}{8}$  chance of winning \$4, and a  $\frac{1}{8}$  chance of winning \$12. The ticket costs \$2. What is the expected net gain or loss from one ticket?

WORK SPACE

ANSWER \_\_\_\_\_

- 2** A study app gives  $X$  badges in one session.  $X = 0, 1$ , or  $3$  with probabilities  $\frac{1}{5}, \frac{1}{2}$ , and  $\frac{3}{10}$ . If you complete 20 sessions, how many badges should you expect to earn?

WORK SPACE

ANSWER \_\_\_\_\_

## CHALLENGE One step up

### CHALLENGE

A school club must choose one prize wheel. Wheel A gives  $X = 0, 4, 8$  with probabilities  $\frac{1}{2}, \frac{1}{4}, \frac{1}{4}$ . Wheel B gives  $X = 1, 5, 9$  with probabilities  $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$ . Which wheel has the greater expected prize, and by how much? Explain using expected values.

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## PART 1 • PRACTICE

| # | PROBLEM                                                                                                              | ANSWER                                                   |
|---|----------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------|
| 1 | A game gives X points with this distribution: $X = 0$ or $6$ , and $P(X) = 3/4$ or $1/4$ . Find $E(X)$ .             | <b>3/2 points</b>                                        |
| 2 | A player can gain or lose points. $X = -4, 2$ , or $8$ , with probabilities $2/5, 1/5$ , and $2/5$ . Find $E(X)$ .   | <b>2 points</b>                                          |
| 3 | Is this a valid probability distribution? $X = 2, 4, 6$ and $P(X) = 1/5, 3/10, 1/2$ . Show why.                      | <b>Yes, because <math>1/5 + 3/10 + 1/2 = 1</math>.</b>   |
| 4 | A distribution has $X = 0, 5, 10$ and $P(X) = 1/8, p, 3/8$ . Find $p$ , then find $E(X)$ .                           | <b><math>p = 1/2</math> and <math>E(X) = 25/4</math></b> |
| 5 | For $X = 2, 4, 6, 8$ , the probabilities are $3/10, 1/5, 3/10, 1/5$ . Find $P(X \text{ is at least } 4)$ .           | <b>7/10</b>                                              |
| 6 | A practice challenge awards X points. $X = 2, 5, 9$ and $P(X) = 1/3, 1/2, 1/6$ . Find the expected number of points. | <b>14/3 points</b>                                       |

## PART 2 • APPLY IT

- Expected net gain: \$1** The expected prize is  $0(1/2) + 4(3/8) + 12(1/8) = \$3$ . Subtract the \$2 ticket cost to get an expected net gain of \$1.
- 28 badges** The expected badges per session are  $0(1/5) + 1(1/2) + 3(3/10) = 7/5$ . Multiply  $7/5$  by 20 sessions to get 28 badges.

## CHALLENGE

**Wheel B has the greater expected prize.  $E(A) = 0(1/2) + 4(1/4) + 8(1/4) = 3$ , and  $E(B) = 1(1/4) + 5(1/2) + 9(1/4) = 5$ , so Wheel B is greater by 2.**

Accept equivalent fractions and decimals, such as 1.5 for  $3/2$  and 6.25 for  $25/4$ . For explanation items, require correct addition of probabilities or correct expected-value calculations.