LESSON MAKER PRO

Chance and Choice

Find expected values and use them to compare outcomes.

HIGH SCHOOL · MA.912.DP.6

MA.912.DP.6 "Use probability distributions to solve problems."

NAME _____ DATE _____

EXPECTED VALUE

A probability distribution lists each possible outcome and its probability. To find the expected value, multiply each outcome by its probability and add the products, then use that long-run average to answer the question.

WORKED EXAMPLE

A game has score X with this distribution: X = 13, 17, 21 and P(X) = 2/7, 3/7, 2/7. Find the expected score.

1. E(X) = 13(2/7) + 17(3/7) + 21(2/7)
2. E(X) = 26/7 + 51/7 + 42/7
3. E(X) = 119/7
4. E(X) = 17

Answer: 17 points

PRACTICE 6 problems

1 A game gives X points with this distribution: X = 0 or 6, and P(X) = 3/4 or 1/4. Find E(X).

ANSWER _____

2 A player can gain or lose points. X = -4, 2, or 8, with probabilities 2/5, 1/5, and 2/5. Find E(X).

ANSWER _____

3 Is this a valid probability distribution? X = 2, 4, 6 and P(X) = 1/5, 3/10, 1/2. Show why.

ANSWER _____

4 A distribution has X = 0, 5, 10 and P(X) = 1/8, p, 3/8. Find p, then find E(X).

ANSWER _____

5 For X = 2, 4, 6, 8, the probabilities are 3/10, 1/5, 3/10, 1/5. Find P(X is at least 4).

ANSWER _____

6 A practice challenge awards X points. X = 2, 5, 9 and P(X) = 1/3, 1/2, 1/6. Find the expected number of points.

ANSWER _____

APPLY IT Show your work

1 At an esports club raffle, a ticket has a 1/2 chance of winning $0, a 3/8 chance of winning $4, and a 1/8 chance of winning $12. The ticket costs $2. What is the expected net gain or loss from one ticket?

WORK SPACE

ANSWER _____

2 A study app gives X badges in one session. X = 0, 1, or 3 with probabilities 1/5, 1/2, and 3/10. If you complete 20 sessions, how many badges should you expect to earn?

WORK SPACE

ANSWER _____

CHALLENGE One step up

A school club must choose one prize wheel. Wheel A gives X = 0, 4, 8 with probabilities 1/2, 1/4, 1/4. Wheel B gives X = 1, 5, 9 with probabilities 1/4, 1/2, 1/4. Which wheel has the greater expected prize, and by how much? Explain using expected values.

WORK SPACE AND ANSWER

Chance and Choice

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	A game gives X points with this distribution: X = 0 or 6, and P(X) = 3/4 or 1/4. Find E(X).	**3/2 points**
2	A player can gain or lose points. X = -4, 2, or 8, with probabilities 2/5, 1/5, and 2/5. Find E(X).	**2 points**
3	Is this a valid probability distribution? X = 2, 4, 6 and P(X) = 1/5, 3/10, 1/2. Show why.	**Yes, because 1/5 + 3/10 + 1/2 = 1.**
4	A distribution has X = 0, 5, 10 and P(X) = 1/8, p, 3/8. Find p, then find E(X).	**p = 1/2 and E(X) = 25/4**
5	For X = 2, 4, 6, 8, the probabilities are 3/10, 1/5, 3/10, 1/5. Find P(X is at least 4).	**7/10**
6	A practice challenge awards X points. X = 2, 5, 9 and P(X) = 1/3, 1/2, 1/6. Find the expected number of points.	**14/3 points**

PART 2 · APPLY IT

1. **Expected net gain: $1** The expected prize is 0(1/2) + 4(3/8) + 12(1/8) = $3. Subtract the $2 ticket cost to get an expected net gain of $1.
2. **28 badges** The expected badges per session are 0(1/5) + 1(1/2) + 3(3/10) = 7/5. Multiply 7/5 by 20 sessions to get 28 badges.

CHALLENGE

Wheel B has the greater expected prize. E(A) = 0(1/2) + 4(1/4) + 8(1/4) = 3, and E(B) = 1(1/4) + 5(1/2) + 9(1/4) = 5, so Wheel B is greater by 2.

Accept equivalent fractions and decimals, such as 1.5 for 3/2 and 6.25 for 25/4. For explanation items, require correct addition of probabilities or correct expected-value calculations.