

Function Shift Detectives

Use matching table values to identify k and the transformation.

HIGH SCHOOL • MA.912.F.2.3

MA.912.F.2.3 "Given the graph or table of $f(x)$ and the graph or table of $f(x)+k$, $kf(x)$, $f(kx)$ and $f(x+k)$, state the type of transformation and find the value of the real number k ."

NAME _____

DATE _____

COMPARE WHAT CHANGES

Compare matching inputs and outputs in the two tables. In $f(x)+k$ and $kf(x)$, the x -values stay the same. In $f(kx)$ and $f(x+k)$, track how the x -values change: $f(kx)$ scales horizontally by $1/|k|$, while $f(x+k)$ shifts left when k is positive and right when k is negative.

WORKED EXAMPLE

The tables show $g(x)=f(x)+k$. $f(x)$: $x \mid 21 \mid 23 \mid y \mid -17 \mid 19$ $g(x)$: $x \mid 21 \mid 23 \mid y \mid -8 \mid 28$ State the transformation and find k .

1. Use matching x -values: $-8 - (-17) = 9$.
2. Check the other pair: $28 - 19 = 9$.
3. So $g(x)=f(x)+9$.
4. Adding 9 shifts the graph up 9 units.

Answer: Vertical shift up 9 units; $k=9$.

PRACTICE 6 problems

- 1** The tables show $g(x)=f(x)+k$. $f(x)$: $x \mid -3 \mid 0 \mid 3 \mid y \mid 2 \mid -2 \mid 7$ $g(x)$: $x \mid -3 \mid 0 \mid 3 \mid y \mid 10 \mid 6 \mid 15$ State the transformation and find k .

ANSWER _____

- 2** The tables show $h(x)=kf(x)$. $f(x)$: $x \mid -2 \mid 1 \mid 4 \mid y \mid 3 \mid -1 \mid 5$ $h(x)$: $x \mid -2 \mid 1 \mid 4 \mid y \mid -6 \mid 2 \mid -10$ State the transformation and find k .

ANSWER _____

- 3** The tables show $q(x)=f(kx)$. $f(x)$: $x \mid -15 \mid -3 \mid 6 \mid y \mid 4 \mid 8 \mid -1$ $q(x)$: $x \mid -5 \mid -1 \mid 2 \mid y \mid 4 \mid 8 \mid -1$ State the transformation and find k .

ANSWER _____

- 4** The tables show $r(x)=f(x+k)$. $f(x)$: $x \mid -11 \mid -5 \mid 1 \mid y \mid 7 \mid 0 \mid -6$ $r(x)$: $x \mid -13 \mid -7 \mid -1 \mid y \mid 7 \mid 0 \mid -6$ State the transformation and find k .

ANSWER _____

- 5** The tables show $s(x)=f(x)+k$. $f(x)$: $x \mid -6 \mid 2 \mid 8 \mid y \mid 10 \mid 4 \mid -1$ $s(x)$: $x \mid -6 \mid 2 \mid 8 \mid y \mid 6 \mid 0 \mid -5$ State the transformation and find k .

ANSWER _____

- 6** The tables show $p(x)=f(kx)$. $f(x)$: $x \mid -8 \mid -2 \mid 4 \mid y \mid 1 \mid 6 \mid -3$ $p(x)$: $x \mid 8 \mid 2 \mid -4 \mid y \mid 1 \mid 6 \mid -3$ State the transformation and find k .

ANSWER _____

APPLY IT Show your work

- 1** A school esports league tracks a player's score with $f(x)$, where x is the number of rounds played. During a special event, the score is modeled by $g(x)=f(x)+k$. $f(x)$: $x \mid 1 \mid 2 \mid 3 \mid y \mid 14 \mid 20 \mid 26$ $g(x)$: $x \mid 1 \mid 2 \mid 3 \mid y \mid 18 \mid 24 \mid 30$ What transformation changed the score model, and what is k ?

WORK SPACE

ANSWER _____

- 2** A student edits a slow-motion skate video. In the original video, $f(x)$ gives the height of an on-screen graphic at x seconds. In the slow-motion edit, $h(x)=f(kx)$. $f(x)$: $x \mid 2 \mid 6 \mid 10 \mid y \mid 3 \mid 7 \mid 11$ $h(x)$: $x \mid 4 \mid 12 \mid 20 \mid y \mid 3 \mid 7 \mid 11$ What transformation changed the timing, and what is k ?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

The table for u is made from the table for f using exactly one of these forms: $f(x)+k$, $kf(x)$, $f(kx)$, or $f(x+k)$. $f(x)$: $x \mid -6 \mid 2 \mid 8 \mid y \mid 4 \mid -2 \mid 5$ $u(x)$: $x \mid -3 \mid 1 \mid 4 \mid y \mid 4 \mid -2 \mid 5$ Identify the form, state the transformation, find k , and explain how the tables prove your answer.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	The tables show $g(x)=f(x)+k$. $f(x)$: $x \mid -3 \mid 0 \mid 3 \mid y \mid 2 \mid -2 \mid 7$ $g(x)$: $x \mid -3 \mid 0 \mid 3 \mid y \mid 10 \mid 6 \mid 15$ State the transformation and find k .	Vertical shift up 8 units; $k=8$.
2	The tables show $h(x)=kf(x)$. $f(x)$: $x \mid -2 \mid 1 \mid 4 \mid y \mid 3 \mid -1 \mid 5$ $h(x)$: $x \mid -2 \mid 1 \mid 4 \mid y \mid -6 \mid 2 \mid -10$ State the transformation and find k .	Reflection across the x-axis and vertical stretch by a factor of 2; $k=-2$.
3	The tables show $q(x)=f(kx)$. $f(x)$: $x \mid -15 \mid -3 \mid 6 \mid y \mid 4 \mid 8 \mid -1$ $q(x)$: $x \mid -5 \mid -1 \mid 2 \mid y \mid 4 \mid 8 \mid -1$ State the transformation and find k .	Horizontal compression by a factor of $1/3$; $k=3$.
4	The tables show $r(x)=f(x)+k$. $f(x)$: $x \mid -11 \mid -5 \mid 1 \mid y \mid 7 \mid 0 \mid -6$ $r(x)$: $x \mid -13 \mid -7 \mid -1 \mid y \mid 7 \mid 0 \mid -6$ State the transformation and find k .	Horizontal shift left 2 units; $k=2$.
5	The tables show $s(x)=f(x)+k$. $f(x)$: $x \mid -6 \mid 2 \mid 8 \mid y \mid 10 \mid 4 \mid -1$ $s(x)$: $x \mid -6 \mid 2 \mid 8 \mid y \mid 6 \mid 0 \mid -5$ State the transformation and find k .	Vertical shift down 4 units; $k=-4$.
6	The tables show $p(x)=f(kx)$. $f(x)$: $x \mid -8 \mid -2 \mid 4 \mid y \mid 1 \mid 6 \mid -3$ $p(x)$: $x \mid 8 \mid 2 \mid -4 \mid y \mid 1 \mid 6 \mid -3$ State the transformation and find k .	Reflection across the y-axis; $k=-1$.

PART 2 • APPLY IT

- Vertical shift up 4 units; $k=4$.** Compare scores after the same number of rounds. Each $g(x)$ value is 4 more than the matching $f(x)$ value.
- Horizontal stretch by a factor of 2; $k=1/2$.** Each matching output occurs at an input that is twice as large in h . Therefore $h(x)=f((1/2)x)$, which is a horizontal stretch by 2.

CHALLENGE

$u(x)=f(2x)$, so the transformation is a horizontal compression by a factor of $1/2$ and $k=2$. Each u input is half of the f input paired with the same output.

Accept equivalent descriptions such as horizontal shrink by $1/3$ for horizontal compression by a factor of $1/3$. For negative vertical factors, students should identify both the reflection across the x-axis and the vertical stretch.