LESSON MAKER PRO

Flip the Function

Restrict a domain, then build an inverse.

HIGH SCHOOL · MA.912.F.3.8

MA.912.F.3.8 "Produce an invertible function from a non-invertible function by restricting the domain."

NAME ______________________ DATE ____________

MAKE IT ONE-TO-ONE

A function is invertible when each output matches only one input. For a parabola or an absolute-value function, restrict the domain to one side of its vertex or corner, then solve y=... for y to write the inverse.

WORKED EXAMPLE

Restrict F(x)=(x+2)^2+8 to x<=-2. Find F^-1(x).

1. Write y=(x+2)^2+8, with x<=-2.
2. Swap x and y: x=(y+2)^2+8.
3. Solve for y. Because y<=-2, y=-2-sqrt(x-8).

Answer: Restricted function: F(x)=(x+2)^2+8, x<=-2. F^-1(x)=-2-sqrt(x-8), x>=8.

PRACTICE

6 problems

1. Restrict f(x)=x^2 to x>=0. Write f^-1(x) and its domain.

   ANSWER ______________________

2. Restrict g(x)=x^2+5 to x<=0. Write g^-1(x) and its domain.

   ANSWER ______________________

3. Restrict h(x)=(x-3)^2 to x>=3. Write h^-1(x) and its domain.

   ANSWER ______________________

4. Restrict p(x)=(x+4)^2-1 to x<=-4. Write p^-1(x) and its domain.

   ANSWER ______________________

5. The function q(x)=|x-6| is non-invertible. Restrict its domain so q is invertible and its inverse has a positive rule. Write the restriction and q^-1(x).

   ANSWER ______________________

6. Restrict r(x)=(x-1)^2+9 to x<=1. Write r^-1(x) and its domain.

   ANSWER ______________________

APPLY IT Show your work

1 During a theater-tech rehearsal, a moving spotlight has height $h(t)=-(t-5)^2+30$, where t is time in seconds and $0<=t<=10$. The stage manager wants time as a function of height while the spotlight is rising. What domain should be used for h, and what is $h^{-1}(x)$, including its domain?

WORK SPACE

ANSWER ______________________

2 A game developer models an avatar's jump height with $P(t)=-(t-3)^2+10$, where t is time in seconds and $0<=t<=6$. For the descending part of the jump, make P invertible and write the inverse function, including its domain.

WORK SPACE

ANSWER ______________________

CHALLENGE One step up

CHALLENGE

For $f(x)=x^2-6x+5$, find two different domain restrictions that make f invertible. Write the inverse function for each restriction, including each inverse domain.

____

____

ANSWER KEY

Flip the Function

HIGH SCHOOL · MA.912.F.3.8

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	Restrict f(x)=x^2 to x>=0. Write f^-1(x) and its domain.	**f^-1(x)=sqrt(x), x>=0.**
2	Restrict g(x)=x^2+5 to x<=0. Write g^-1(x) and its domain.	**g^-1(x)=-sqrt(x-5), x>=5.**
3	Restrict h(x)=(x-3)^2 to x>=3. Write h^-1(x) and its domain.	**h^-1(x)=sqrt(x)+3, x>=0.**
4	Restrict p(x)=(x+4)^2-1 to x<=-4. Write p^-1(x) and its domain.	**p^-1(x)=-sqrt(x+1)-4, x>=-1.**
5	The function q(x)=|x-6| is non-invertible. Restrict its domain so q is invertible and its inverse has a positive rule. Write the restriction and q^-1(x).	**Restrict q to x>=6. q^-1(x)=x+6, x>=0.**
6	Restrict r(x)=(x-1)^2+9 to x<=1. Write r^-1(x) and its domain.	**r^-1(x)=1-sqrt(x-9), x>=9.**

PART 2 · APPLY IT

1. **Use 0<=t<=5. h^-1(x)=5-sqrt(30-x), with 5<=x<=30.** The spotlight rises until its maximum height at t=5, so keep the left branch. Solve x=-(t-5)^2+30 and choose the negative square-root branch.
2. **Use 3<=t<=6. P^-1(x)=3+sqrt(10-x), with 1<=x<=10.** The avatar descends after the vertex at t=3, so keep the right branch. Solve x=-(t-3)^2+10 and choose the positive square-root branch.

CHALLENGE

If x>=3, then f^-1(x)=3+sqrt(x+4), with x>=-4. If x<=3, then f^-1(x)=3-sqrt(x+4), with x>=-4.

Accept equivalent inverse forms, such as -sqrt(x-5) instead of 0-sqrt(x-5). Students must pair each inverse rule with the correct restricted branch and inverse domain.