

Cone Slice Detectives

Use angles to identify each cross section.

HIGH SCHOOL • MA.912.GR.7.1

MA.912.GR.7.1 "Given a conic section, describe how it can result from the slicing of two cones."

NAME _____

DATE _____

COMPARE THE ANGLES

Let a be the angle between the axis and a generator of a double cone, and let p be the angle between the axis and the cutting plane. If $p = 90^\circ$, the slice is a circle; if $a < p < 90^\circ$, it is an ellipse; if $p = a$, it is a parabola; if $p < a$, it is a hyperbola. Assume the plane does not pass through the vertex.

WORKED EXAMPLE

A double cone has $a = 37^\circ$. A cutting plane has $p = 64^\circ$. Name the cross section.

1. Compare the angles: $37^\circ < 64^\circ < 90^\circ$.
2. The plane angle is greater than a but less than 90° .
3. That angle range makes an ellipse.

Answer: ellipse

PRACTICE 5 problems

- 1** A double cone has $a = 28^\circ$. A slice is a circle. What is p ?

ANSWER _____

- 2** A double cone has $a = 45^\circ$. A slice is a parabola. What is p ?

ANSWER _____

- 3** A double cone has $a = 52^\circ$. A slice is a hyperbola. Which value of p works: 31° , 52° , or 78° ?

ANSWER _____

- 4** A double cone has $a = 24^\circ$. Can $p = 63^\circ$ make an ellipse? Write the comparison.

ANSWER _____

- 5** A double cone has $a = 70^\circ$ and $p = 18^\circ$. Name the cross section and describe the cut.

ANSWER _____

APPLY IT Show your work

1

A 3D-printing club models a trophy as a double cone. Its generator angle is $a = 30^\circ$. The club wants a parabolic cross section for a design panel. What angle p should the virtual cutting plane make with the axis?

WORK SPACE

ANSWER _____

2

For a digital art logo, a student uses a double cone with $a = 40^\circ$. Which cutting-plane angle makes a hyperbola: 25° , 40° , 65° , or 90° ?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A double cone has $a = 48^\circ$. Explain why $p = 48^\circ$ makes a parabola, $p = 49^\circ$ makes an ellipse, and $p = 47^\circ$ makes a hyperbola.

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	A double cone has $a = 28^\circ$. A slice is a circle. What is p ?	$p = 90^\circ$
2	A double cone has $a = 45^\circ$. A slice is a parabola. What is p ?	$p = 45^\circ$
3	A double cone has $a = 52^\circ$. A slice is a hyperbola. Which value of p works: 31° , 52° , or 78° ?	31°
4	A double cone has $a = 24^\circ$. Can $p = 63^\circ$ make an ellipse? Write the comparison.	Yes, $24^\circ < 63^\circ < 90^\circ$.
5	A double cone has $a = 70^\circ$ and $p = 18^\circ$. Name the cross section and describe the cut.	hyperbola; the plane cuts both halves of the double cone

PART 2 · APPLY IT

1. **$p = 30^\circ$** A parabola occurs when the cutting plane angle equals the generator angle. Set p equal to 30° .
2. **25°** A hyperbola occurs when p is less than a . Only 25° is less than 40° .

CHALLENGE

Because $p = a = 48^\circ$, the first slice is a parabola. Since $48^\circ < 49^\circ < 90^\circ$, the second slice is an ellipse, and since $47^\circ < 48^\circ$, the third slice is a hyperbola.

Accept equivalent angle comparisons and descriptions of the plane. For a hyperbola, accept "cuts both cones" or "cuts both halves of the double cone."