

Ellipse Equation Lab

Build each equation from the ellipse's key features.

HIGH SCHOOL • MA.912.GR.7.6

MA.912.GR.7.6 "Given a mathematical or real-world context, derive and create the equation of an ellipse using key features."

NAME _____

DATE _____

BUILD THE EQUATION

For an ellipse with center (h,k) , write $(x-h)^2/a^2 + (y-k)^2/b^2 = 1$ when the major axis is horizontal. If the major axis is vertical, place a^2 under the y term and b^2 under the x term. The value a is the center-to-vertex distance, b is the center-to-covertex distance, and $c^2 = a^2 - b^2$ when foci are given.

WORKED EXAMPLE

Write the equation of the ellipse with center $(5,-7)$, horizontal vertices $(-4,-7)$ and $(14,-7)$, and covertices $(5,-3)$ and $(5,-11)$.

1. The center is $(h,k) = (5,-7)$.
2. The horizontal vertex distance is $a = 9$, so $a^2 = 81$.
3. The vertical covertex distance is $b = 4$, so $b^2 = 16$.
4. Use the horizontal major-axis form.

Answer: $(x-5)^2/81 + (y+7)^2/16 = 1$

PRACTICE 6 problems

- 1** Write the equation of the ellipse with center $(2,-1)$, horizontal vertices $(-6,-1)$ and $(10,-1)$, and covertices $(2,-4)$ and $(2,2)$.

ANSWER _____

- 2** Write the equation of the ellipse with center $(-3,5)$, vertical vertices $(-3,-4)$ and $(-3,14)$, and covertices $(-9,5)$ and $(3,5)$.

ANSWER _____

- 3** Write the equation of the ellipse with center $(0,0)$, horizontal vertices $(-12,0)$ and $(12,0)$, and covertices $(0,-5)$ and $(0,5)$.

ANSWER _____

- 4** Write the equation of the ellipse with center $(1,2)$, vertical vertices $(1,14)$ and $(1,-10)$, and foci $(1,10)$ and $(1,-6)$.

ANSWER _____

- 5** Write the equation of the ellipse with center $(-7,-2)$, a vertical major axis of length 20, and a minor axis of length 16.

ANSWER _____

- 6** Write the equation of the ellipse with center $(3,4)$, horizontal vertices $(-9,4)$ and $(15,4)$, and foci $(-6,4)$ and $(12,4)$.

ANSWER _____

APPLY IT Show your work

- 1** A game club is designing an elliptical arena on a coordinate map. The arena is centered at $(6, -2)$, has a horizontal width of 20 units, and has a vertical height of 12 units. Write the equation of its boundary.

WORK SPACE

ANSWER _____

- 2** A student film crew plans an elliptical light zone for an outdoor scene. The zone has center $(-4, 3)$, vertical vertices $(-4, 12)$ and $(-4, -6)$, and foci $(-4, 9)$ and $(-4, -3)$. Write the equation of the light-zone boundary.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

An ellipse is centered at $(0, 0)$, has foci $(-8, 0)$ and $(8, 0)$, and passes through $(0, 15)$. Write its equation and explain how you found both denominators.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Write the equation of the ellipse with center (2,-1), horizontal vertices (-6,-1) and (10,-1), and covertices (2,-4) and (2,2).	$(x-2)^2/64 + (y+1)^2/9 = 1$
2	Write the equation of the ellipse with center (-3,5), vertical vertices (-3,-4) and (-3,14), and covertices (-9,5) and (3,5).	$(x+3)^2/36 + (y-5)^2/81 = 1$
3	Write the equation of the ellipse with center (0,0), horizontal vertices (-12,0) and (12,0), and covertices (0,-5) and (0,5).	$x^2/144 + y^2/25 = 1$
4	Write the equation of the ellipse with center (1,2), vertical vertices (1,14) and (1,-10), and foci (1,10) and (1,-6).	$(x-1)^2/80 + (y-2)^2/144 = 1$
5	Write the equation of the ellipse with center (-7,-2), a vertical major axis of length 20, and a minor axis of length 16.	$(x+7)^2/64 + (y+2)^2/100 = 1$
6	Write the equation of the ellipse with center (3,4), horizontal vertices (-9,4) and (15,4), and foci (-6,4) and (12,4).	$(x-3)^2/144 + (y-4)^2/63 = 1$

PART 2 • APPLY IT

1. $(x-6)^2/100 + (y+2)^2/36 = 1$ The horizontal semi-major axis is $a = 10$ and the vertical semi-minor axis is $b = 6$. Substitute the center and squared axis lengths into the horizontal major-axis form.
2. $(x+4)^2/45 + (y-3)^2/81 = 1$ The vertical semi-major axis is $a = 9$, and the focal distance is $c = 6$. Use $b^2 = a^2 - c^2 = 81 - 36 = 45$, then use the vertical major-axis form.

CHALLENGE

$x^2/289 + y^2/225 = 1$. The point (0,15) is 17 units from each focus, so $2a = 34$ and $a = 17$. Since $c = 8$, $b^2 = a^2 - c^2 = 289 - 64 = 225$.

Accept equivalent equations with terms reversed, such as placing the y term first. Require squared denominators and correct signs inside each grouped coordinate term.