

Ellipse Mission Map

Find features, graph boundaries, and solve intersections.

HIGH SCHOOL • MA.912.GR.7.7

MA.912.GR.7.7 "Graph and solve mathematical and real-world problems that are modeled with an equation of an ellipse. Determine and interpret key features in terms of the context."

NAME _____

DATE _____

READ THE ELLIPSE

In $(x-h)^2/a^2 + (y-k)^2/b^2 = 1$, the center is (h,k) . The larger denominator shows the direction of the major axis, and the square roots of the denominators give the distances from the center to the vertices and covertices.

WORKED EXAMPLE

Graph the ellipse and list its center, vertices, covertices, and foci: $(x-8)^2/100 + (y+6)^2/49 = 1$.

1. The center is $(8,-6)$. Since 100 is larger than 49, the major axis is horizontal.
2. $a = 10$ and $b = 7$, so the vertices are 10 units left and right of the center.
3. The vertices are $(-2,-6)$ and $(18,-6)$, and the covertices are $(8,1)$ and $(8,-13)$.
4. $c^2 = 100 - 49 = 51$, so the foci are $(8-\sqrt{51},-6)$ and $(8+\sqrt{51},-6)$.

Answer: Center: $(8,-6)$. Vertices: $(-2,-6)$, $(18,-6)$. Covertices: $(8,1)$, $(8,-13)$. Foci: $(8-\sqrt{51},-6)$, $(8+\sqrt{51},-6)$.

PRACTICE 6 problems

- 1** For $(x+2)^2/25 + (y-5)^2/9 = 1$, state the center, the direction of the major axis, and the lengths of the major and minor axes.

ANSWER _____

- 2** List the vertices and covertices of $(x-3)^2/36 + (y+4)^2/4 = 1$.

ANSWER _____

- 3** Find the foci of $(x+1)^2/9 + (y-2)^2/64 = 1$.

ANSWER _____

- 4** Write an equation of the ellipse with center $(-1,4)$, horizontal vertices $(-6,4)$ and $(4,4)$, and covertices $(-1,1)$ and $(-1,7)$.

ANSWER _____

- 5** Solve the system $y = 3$ and $(x-5)^2/16 + (y-3)^2/36 = 1$. Give both intersection points.

ANSWER _____

- 6** Solve the system $x = -2$ and $(x+2)^2/4 + (y-1)^2/25 = 1$. Give both intersection points.

ANSWER _____

APPLY IT Show your work

- 1** At a school festival, an elliptical light projection is modeled in meters by $(x-2)^2/81 + (y+1)^2/25 = 1$. Find the center of the light zone, its farthest east, west, north, and south boundary points, and its total width and height.

WORK SPACE

ANSWER _____

- 2** A game design class models a virtual arena boundary by $x^2/36 + (y-4)^2/25 = 1$. A player travels along the path $y = 1$. Where does the path enter and leave the arena?

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

An ellipse has vertices $(-5,-1)$ and $(9,-1)$ and foci $(-1,-1)$ and $(5,-1)$. Write its equation and explain how the foci help determine the missing denominator.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	For $(x+2)^2/25 + (y-5)^2/9 = 1$, state the center, the direction of the major axis, and the lengths of the major and minor axes.	Center: (-2,5). Major axis: horizontal. Major axis length: 10. Minor axis length: 6.
2	List the vertices and covertices of $(x-3)^2/36 + (y+4)^2/4 = 1$.	Vertices: (-3,-4), (9,-4). Covertices: (3,-6), (3,-2).
3	Find the foci of $(x+1)^2/9 + (y-2)^2/64 = 1$.	Foci: (-1,2-$\sqrt{55}$), (-1,2+$\sqrt{55}$).
4	Write an equation of the ellipse with center (-1,4), horizontal vertices (-6,4) and (4,4), and covertices (-1,1) and (-1,7).	$(x+1)^2/25 + (y-4)^2/9 = 1$
5	Solve the system $y = 3$ and $(x-5)^2/16 + (y-3)^2/36 = 1$. Give both intersection points.	(1,3) and (9,3)
6	Solve the system $x = -2$ and $(x+2)^2/4 + (y-1)^2/25 = 1$. Give both intersection points.	(-2,-4) and (-2,6)

PART 2 • APPLY IT

- Center: (2,-1). East: (11,-1). West: (-7,-1). North: (2,4). South: (2,-6). Width: 18 m. Height: 10 m.** The center is (2,-1), with horizontal radius 9 and vertical radius 5. Add and subtract each radius from the matching center coordinate.
- The path enters and leaves at (-24/5,1) and (24/5,1).** Substitute $y = 1$ to get $x^2/36 + 9/25 = 1$. Then $x^2/36 = 16/25$, so $x =$ plus or minus $24/5$.

CHALLENGE

The equation is $(x-2)^2/49 + (y+1)^2/40 = 1$. The center is (2,-1), so $a = 7$ and $c = 3$. Since $c^2 = a^2 - b^2$, $b^2 = 49 - 9 = 40$.

Accept equivalent equation forms with the terms reversed. Accept foci and intersection points in either order, and accept plus or minus notation when it represents both points correctly.