

Hyperbola Feature Forge

Build each equation from the clues.

HIGH SCHOOL • MA.912.GR.7.8

MA.912.GR.7.8 "Given a mathematical or real-world context, derive and create the equation of a hyperbola using key features."

NAME _____

DATE _____

TURN FEATURES INTO EQUATIONS

A hyperbola with a horizontal transverse axis uses $(x-h)^2/a^2 - (y-k)^2/b^2 = 1$. A vertical transverse axis switches the x and y terms, and the center is (h,k) , while a is the distance to a vertex and b can come from conjugate-axis endpoints, asymptote slopes, or $c^2 = a^2 + b^2$.

WORKED EXAMPLE

A horizontal hyperbola has center $(12, -17)$, $a = 9$, and $b = 15$. Write its equation.

1. Start with $(x-h)^2/a^2 - (y-k)^2/b^2 = 1$.
2. Use $h = 12$ and $k = -17$.
3. Use $a^2 = 81$ and $b^2 = 225$.

Answer: $(x-12)^2/81 - (y+17)^2/225 = 1$

PRACTICE 6 problems

- 1** A hyperbola has center $(6, -1)$, vertices $(4, -1)$ and $(8, -1)$, and conjugate-axis endpoints $(6, 4)$ and $(6, -6)$. Write its equation.

ANSWER _____

- 2** A hyperbola has center $(-3, 2)$, vertices $(-3, 7)$ and $(-3, -3)$, and conjugate-axis endpoints $(1, 2)$ and $(-7, 2)$. Write its equation.

ANSWER _____

- 3** A horizontal hyperbola has vertices $(-5, 0)$ and $(5, 0)$ and foci $(-13, 0)$ and $(13, 0)$. Write its equation.

ANSWER _____

- 4** A horizontal hyperbola has center $(2, 1)$, vertices $(-2, 1)$ and $(6, 1)$, and asymptotes $y - 1 = \frac{1}{2}(x - 2)$ and $y - 1 = -\frac{1}{2}(x - 2)$. Write its equation.

ANSWER _____

- 5** A vertical hyperbola has center $(3, -2)$, one vertex at $(3, 2)$, and passes through $(6, 6)$. Write its equation.

ANSWER _____

- 6** A vertical hyperbola has vertices $(1, -2)$ and $(1, -6)$ and asymptotes $y + 4 = \frac{3}{2}(x - 1)$ and $y + 4 = -\frac{3}{2}(x - 1)$. Write its equation.

ANSWER _____

APPLY IT Show your work

- 1** At a music festival, two signal receivers are at $(0,0)$ and $(10,0)$. A location boundary consists of all points whose distances from the two receivers differ by 6 units. Write the equation of the boundary.

WORK SPACE

ANSWER _____

- 2** A phone app models a connection boundary using cell towers at $(4,1)$ and $(4,13)$. The boundary contains points whose distances from the towers differ by 8 units. Write the equation of the boundary.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A horizontal hyperbola has center $(2,-1)$, foci $(-3,-1)$ and $(7,-1)$, and asymptotes $y+1 = \frac{4}{3}(x-2)$ and $y+1 = -\frac{4}{3}(x-2)$. Write its equation and explain how you found a and b .

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	A hyperbola has center (6,-1), vertices (4,-1) and (8,-1), and conjugate-axis endpoints (6,4) and (6,-6). Write its equation.	$(x-6)^2/4 - (y+1)^2/25 = 1$
2	A hyperbola has center (-3,2), vertices (-3,7) and (-3,-3), and conjugate-axis endpoints (1,2) and (-7,2). Write its equation.	$(y-2)^2/25 - (x+3)^2/16 = 1$
3	A horizontal hyperbola has vertices (-5,0) and (5,0) and foci (-13,0) and (13,0). Write its equation.	$x^2/25 - y^2/144 = 1$
4	A horizontal hyperbola has center (2,1), vertices (-2,1) and (6,1), and asymptotes $y-1 = 1/2(x-2)$ and $y-1 = -1/2(x-2)$. Write its equation.	$(x-2)^2/16 - (y-1)^2/4 = 1$
5	A vertical hyperbola has center (3,-2), one vertex at (3,2), and passes through (6,6). Write its equation.	$(y+2)^2/16 - (x-3)^2/3 = 1$
6	A vertical hyperbola has vertices (1,-2) and (1,-6) and asymptotes $y+4 = 3/2(x-1)$ and $y+4 = -3/2(x-1)$. Write its equation.	$(y+4)^2/4 - (x-1)^2/(16/9) = 1$

PART 2 • APPLY IT

1. $(x-5)^2/9 - y^2/16 = 1$ The receivers are the foci, so the center is (5,0) and $c = 5$. The distance difference is $2a = 6$, so $a = 3$ and $b^2 = 25 - 9 = 16$.
2. $(y-7)^2/16 - (x-4)^2/20 = 1$ The center is the midpoint, (4,7), and $c = 6$. Since $2a = 8$, $a = 4$, so $b^2 = 36 - 16 = 20$.

CHALLENGE

The foci give $c = 5$. Since $b/a = 4/3$ and $c^2 = a^2 + b^2$, $a = 3$ and $b = 4$, so $(x-2)^2/9 - (y+1)^2/16 = 1$.

Accept algebraically equivalent standard-form equations, including forms that rewrite division by $16/9$ as multiplication by $9/16$. For feature-based items, require the correct center, orientation, and squared values of a and b .