LESSON MAKER PRO

Hyperbola Feature Forge

Build each equation from the clues.

HIGH SCHOOL · MA.912.GR.7.8

MA.912.GR.7.8 "Given a mathematical or real-world context, derive and create the equation of a hyperbola using key features."

NAME _____ DATE _____

TURN FEATURES INTO EQUATIONS

A hyperbola with a horizontal transverse axis uses $(x-h)^2/a^2 - (y-k)^2/b^2 = 1$. A vertical transverse axis switches the x and y terms, and the center is (h,k), while a is the distance to a vertex and b can come from conjugate-axis endpoints, asymptote slopes, or $c^2 = a^2 + b^2$.

WORKED EXAMPLE

A horizontal hyperbola has center (12,-17), a = 9, and b = 15. Write its equation.

1. Start with $(x-h)^2/a^2 - (y-k)^2/b^2 = 1$.
2. Use h = 12 and k = -17.
3. Use $a^2 = 81$ and $b^2 = 225$.

Answer: $(x-12)^2/81 - (y+17)^2/225 = 1$

PRACTICE 6 problems

1 A hyperbola has center (6,-1), vertices (4,-1) and (8,-1), and conjugate-axis endpoints (6,4) and (6,-6). Write its equation.

ANSWER _____

2 A hyperbola has center (-3,2), vertices (-3,7) and (-3,-3), and conjugate-axis endpoints (1,2) and (-7,2). Write its equation.

ANSWER _____

3 A horizontal hyperbola has vertices (-5,0) and (5,0) and foci (-13,0) and (13,0). Write its equation.

ANSWER _____

4 A horizontal hyperbola has center (2,1), vertices (-2,1) and (6,1), and asymptotes y-1 = 1/2(x-2) and y-1 = -1/2(x-2). Write its equation.

ANSWER _____

5 A vertical hyperbola has center (3,-2), one vertex at (3,2), and passes through (6,6). Write its equation.

ANSWER _____

6 A vertical hyperbola has vertices (1,-2) and (1,-6) and asymptotes y+4 = 3/2(x-1) and y+4 = -3/2(x-1). Write its equation.

ANSWER _____

APPLY IT Show your work

1 At a music festival, two signal receivers are at (0,0) and (10,0). A location boundary consists of all points whose distances from the two receivers differ by 6 units. Write the equation of the boundary.

WORK SPACE

ANSWER _____

2 A phone app models a connection boundary using cell towers at (4,1) and (4,13). The boundary contains points whose distances from the towers differ by 8 units. Write the equation of the boundary.

WORK SPACE

ANSWER _____

CHALLENGE One step up

A horizontal hyperbola has center (2,-1), foci (-3,-1) and (7,-1), and asymptotes y+1 = 4/3(x-2) and y+1 = -4/3(x-2). Write its equation and explain how you found a and b.

WORK SPACE AND ANSWER

ANSWER KEY

Hyperbola Feature Forge

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PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	A hyperbola has center (6,-1), vertices (4,-1) and (8,-1), and conjugate-axis endpoints (6,4) and (6,-6). Write its equation.	**$(x-6)^2/4 - (y+1)^2/25 = 1$**
2	A hyperbola has center (-3,2), vertices (-3,7) and (-3,-3), and conjugate-axis endpoints (1,2) and (-7,2). Write its equation.	**$(y-2)^2/25 - (x+3)^2/16 = 1$**
3	A horizontal hyperbola has vertices (-5,0) and (5,0) and foci (-13,0) and (13,0). Write its equation.	**$x^2/25 - y^2/144 = 1$**
4	A horizontal hyperbola has center (2,1), vertices (-2,1) and (6,1), and asymptotes y-1 = 1/2(x-2) and y-1 = -1/2(x-2). Write its equation.	**$(x-2)^2/16 - (y-1)^2/4 = 1$**
5	A vertical hyperbola has center (3,-2), one vertex at (3,2), and passes through (6,6). Write its equation.	**$(y+2)^2/16 - (x-3)^2/3 = 1$**
6	A vertical hyperbola has vertices (1,-2) and (1,-6) and asymptotes y+4 = 3/2(x-1) and y+4 = -3/2(x-1). Write its equation.	**$(y+4)^2/4 - (x-1)^2/(16/9) = 1$**

PART 2 · APPLY IT

1. **$(x-5)^2/9 - y^2/16 = 1$** The receivers are the foci, so the center is (5,0) and c = 5. The distance difference is 2a = 6, so a = 3 and $b^2 = 25 - 9 = 16$.
2. **$(y-7)^2/16 - (x-4)^2/20 = 1$** The center is the midpoint, (4,7), and c = 6. Since 2a = 8, a = 4, so $b^2 = 36 - 16 = 20$.

CHALLENGE

The foci give c = 5. Since b/a = 4/3 and $c^2 = a^2 + b^2$, a = 3 and b = 4, so $(x-2)^2/9 - (y+1)^2/16 = 1$.

Accept algebraically equivalent standard-form equations, including forms that rewrite division by 16/9 as multiplication by 9/16. For feature-based items, require the correct center, orientation, and squared values of a and b.