

Hyperbola Path Finder

Use equations, features, and context to solve hyperbola problems.

HIGH SCHOOL • MA.912.GR.7.9

MA.912.GR.7.9 "Graph and solve mathematical and real-world problems that are modeled with an equation of a hyperbola. Determine and interpret key features in terms of the context."

NAME _____

DATE _____

KEY FEATURES

In a hyperbola, the center is (h,k) . In a horizontal hyperbola, $(x-h)^2/a^2 - (y-k)^2/b^2 = 1$, the vertices are $(h \pm a, k)$ and the asymptotes are $y - k = \pm b/a(x - h)$. For a vertical hyperbola, switch x and y in the squared terms, use vertices $(h, k \pm a)$, and use asymptote slopes $\pm a/b$.

WORKED EXAMPLE

Find the center, vertices, and asymptotes of $(x-31)^2/169 - (y+14)^2/196 = 1$.

1. Read $h = 31$, $k = -14$, $a = 13$, and $b = 14$.
2. The center is $(31, -14)$.
3. The x -term is positive, so the vertices are $(31-13, -14)$ and $(31+13, -14)$.
4. Use $y+14 = \pm 14/13(x-31)$ for the asymptotes.

Answer: Center: $(31, -14)$. Vertices: $(18, -14)$ and $(44, -14)$. Asymptotes: $y+14 = \pm 14/13(x-31)$.

PRACTICE 6 problems

- 1** Find the center, vertices, and asymptotes of $(x+3)^2/16 - (y-2)^2/9 = 1$.

ANSWER _____

- 2** Find the center, vertices, and asymptotes of $(y+4)^2/36 - (x-5)^2/4 = 1$.

ANSWER _____

- 3** Solve the system $x^2/4 - y^2/5 = 1$ and $y = 0$.

ANSWER _____

- 4** Write the equation of a horizontal hyperbola with center $(-6, 7)$, vertices $(-2, 7)$ and $(-10, 7)$, and asymptotes $y - 7 = \pm 3/4(x + 6)$.

ANSWER _____

- 5** Does the point $(4, -5)$ lie on $(x-1)^2/9 - (y+5)^2/16 = 1$?

ANSWER _____

- 6** Solve $(y-6)^2/36 - (x+2)^2/4 = 1$ when $x = -2$.

ANSWER _____

APPLY IT Show your work

- 1** A gaming club splits a \$72 tournament entry fee equally among x players. The cost per player is modeled by $y = 72/x$, where $x > 0$ and $y > 0$. Find the cost per player when 9 players join. Then state the center and asymptotes, and explain what each asymptote means in this context.

WORK SPACE

ANSWER _____

- 2** A 60 GB game update takes y minutes to download at a speed of x GB per minute. The model is $y = 60/x$, where $x > 0$ and $y > 0$. Find the download time at 12 GB per minute. Then state the center and asymptotes and interpret them.

WORK SPACE

ANSWER _____

CHALLENGE One step up

CHALLENGE

A horizontal hyperbola has center $(2, -1)$, vertices $(-2, -1)$ and $(6, -1)$, and positive-slope asymptote $y + 1 = \frac{3}{4}(x - 2)$. Write its equation. Then find all points on the hyperbola where $y = 2$.

PART 1 • PRACTICE

#	PROBLEM	ANSWER
1	Find the center, vertices, and asymptotes of $(x+3)^2/16 - (y-2)^2/9 = 1$.	Center: (-3,2). Vertices: (-7,2) and (1,2). Asymptotes: $y-2 = \pm 3/4(x+3)$.
2	Find the center, vertices, and asymptotes of $(y+4)^2/36 - (x-5)^2/4 = 1$.	Center: (5,-4). Vertices: (5,2) and (5,-10). Asymptotes: $y+4 = \pm 3(x-5)$.
3	Solve the system $x^2/4 - y^2/5 = 1$ and $y = 0$.	(-2,0) and (2,0)
4	Write the equation of a horizontal hyperbola with center (-6,7), vertices (-2,7) and (-10,7), and asymptotes $y-7 = \pm 3/4(x+6)$.	$(x+6)^2/16 - (y-7)^2/9 = 1$
5	Does the point (4,-5) lie on $(x-1)^2/9 - (y+5)^2/16 = 1$?	Yes, (4,-5) lies on the hyperbola.
6	Solve $(y-6)^2/36 - (x+2)^2/4 = 1$ when $x = -2$.	$y = 0$ or $y = 12$

PART 2 • APPLY IT

- The cost is \$8 per player. The center is (0,0), and the asymptotes are $x = 0$ and $y = 0$. Zero players is not possible, and the cost per player approaches but never reaches \$0.** Substitute $x = 9$ into $y = 72/x$ to get $y = 8$. The graph of $y = 72/x$ has the coordinate axes as asymptotes.
- The download time is 5 minutes. The center is (0,0), and the asymptotes are $x = 0$ and $y = 0$. A download speed of 0 is not usable, and download time approaches but never reaches 0 minutes.** Substitute $x = 12$ into $y = 60/x$ to get $y = 5$. The axes are asymptotes because neither speed nor time can equal 0 in the model.

CHALLENGE

Equation: $(x-2)^2/16 - (y+1)^2/9 = 1$. When $y = 2$, the points are $(2-4\sqrt{2}, 2)$ and $(2+4\sqrt{2}, 2)$.

Accept equivalent asymptote equations and equations with terms rearranged. For the challenge, accept $x = 2 \pm 4\sqrt{2}$ in place of the two ordered pairs.