# Induction Launch

*Build a claim from one case to every case.*

HIGH SCHOOL · MA.912.LT.1.3

**MA.912.LT.1.3** "Apply mathematical induction in a variety of applications."

NAME ____________________ DATE ____________

### THE INDUCTION PATTERN

Check the first case. Then assume the claim is true for k and use that assumption to prove it is true for k+1.

### WORKED EXAMPLE

Prove by induction: 13+26+...+13n=13n(n+1)/2, for n>=1.

1. Base: n=1, and 13=13(1)(2)/2.
2. Assume 13+26+...+13k=13k(k+1)/2.
3. Add 13(k+1):
   13k(k+1)/2+13(k+1)=13(k+1)(k+2)/2.

**Answer: True for all n>=1.**

## PRACTICE 5 problems

**1** For P(n): 5+10+...+5n=5n(n+1)/2, write the inductive hypothesis.

**ANSWER** ____________________

**2** Assume 2+4+...+2k=k(k+1). Add the next term and simplify.

**ANSWER** ____________________

**3** Use induction to prove 7^n-1 is divisible by 6 for n>=1.

**ANSWER** ____________________

**4** Use induction to prove 3^n>=2n+1 for n>=0.

**ANSWER** ____________________

**5** Use induction to prove 1+3+...+(2n-1)=n^2 for n>=1.

**ANSWER** ____________________

## APPLY IT Show your work

**1** An esports game awards 14n coins for completing stage n. Use induction to prove that the total after n stages is 7n(n+1) coins.

WORK SPACE

**ANSWER** ____________________

**2** A video-editing contest awards 5n-2 points on day n. Use induction to prove that the total after n days is n(5n+1)/2 points.

WORK SPACE

**ANSWER** ____________________

## CHALLENGE One step up

**CHALLENGE**

Use induction to prove 1^3+2^3+...+n^3=[n(n+1)/2]^2 for n>=1.

## PART 1 · PRACTICE

| # | PROBLEM | ANSWER |
|---|---|---|
| 1 | For P(n): 5+10+...+5n=5n(n+1)/2, write the inductive hypothesis. | **Assume 5+10+...+5k=5k(k+1)/2.** |
| 2 | Assume 2+4+...+2k=k(k+1). Add the next term and simplify. | **k(k+1)+2(k+1)=(k+1)(k+2).** |
| 3 | Use induction to prove 7^n-1 is divisible by 6 for n>=1. | **Base: 7^1-1=6. If 7^k-1=6m, then 7^(k+1)-1=7(7^k-1)+6=6(7m+1).** |
| 4 | Use induction to prove 3^n>=2n+1 for n>=0. | **Base: 3^0=1=2(0)+1. If 3^k>=2k+1, then 3^(k+1)>=6k+3>=2k+3=2(k+1)+1.** |
| 5 | Use induction to prove 1+3+...+(2n-1)=n^2 for n>=1. | **Base: 1=1^2. If the sum through 2k-1 is k^2, adding 2(k+1)-1 gives k^2+2k+1=(k+1)^2.** |

## PART 2 · APPLY IT

1. **Total coins = 7n(n+1).** Base: stage 1 gives 14 coins. If the total through stage k is 7k(k+1), add 14(k+1) to get 7(k+1)(k+2).
2. **Total points = n(5n+1)/2.** Base: day 1 gives 3 points. If the total through day k is k(5k+1)/2, add 5(k+1)-2 to get (k+1)(5k+6)/2.

## CHALLENGE

**Base: 1^3=[1(2)/2]^2. Assume the sum through k is [k(k+1)/2]^2; adding (k+1)^3 gives (k+1)^2[k^2+4k+4]/4=[(k+1)(k+2)/2]^2.**

Accept equivalent algebraic forms. For proof items, require a correct base case, inductive hypothesis, and step from k to k+1.