LESSON MAKER PRO

Induction Launch

Build a claim from one case to every case.

HIGH SCHOOL · MA.912.LT.1.3

MA.912.LT.1.3 "Apply mathematical induction in a variety of applications."

NAME _____ DATE _____

THE INDUCTION PATTERN

Check the first case. Then assume the claim is true for k and use that assumption to prove it is true for k+1.

WORKED EXAMPLE

Prove by induction: $13+26+\ldots+13n=13n(n+1)/2$, for $n>=1$.

1. Base: n=1, and $13=13(1)(2)/2$.
2. Assume $13+26+\ldots+13k=13k(k+1)/2$.
3. Add $13(k+1)$: $13k(k+1)/2+13(k+1)=13(k+1)(k+2)/2$.

Answer: True for all n>=1.

PRACTICE 5 problems

1 For P(n): $5+10+\ldots+5n=5n(n+1)/2$, write the inductive hypothesis.

ANSWER _____

2 Assume $2+4+\ldots+2k=k(k+1)$. Add the next term and simplify.

ANSWER _____

3 Use induction to prove 7^n-1 is divisible by 6 for n>=1.

ANSWER _____

4 Use induction to prove $3^n>=2n+1$ for n>=0.

ANSWER _____

5 Use induction to prove $1+3+\ldots+(2n-1)=n^2$ for n>=1.

ANSWER _____

APPLY IT Show your work

1 An esports game awards 14n coins for completing stage n. Use induction to prove that the total after n stages is 7n(n+1) coins.

WORK SPACE

ANSWER _____

2 A video-editing contest awards 5n-2 points on day n. Use induction to prove that the total after n days is n(5n+1)/2 points.

WORK SPACE

ANSWER _____

CHALLENGE One step up

Use induction to prove $1^3+2^3+...+n^3=[n(n+1)/2]^2$ for n>=1.

WORK SPACE AND ANSWER

ANSWER KEY # Induction Launch

PART 1 · PRACTICE

#	PROBLEM	ANSWER
1	For P(n): 5+10+...+5n=5n(n+1)/2, write the inductive hypothesis.	**Assume 5+10+...+5k=5k(k+1)/2.**
2	Assume 2+4+...+2k=k(k+1). Add the next term and simplify.	**k(k+1)+2(k+1)=(k+1)(k+2).**
3	Use induction to prove 7^n-1 is divisible by 6 for n>=1.	**Base: 7^1-1=6. If 7^k-1=6m, then $7^{(k+1)}$-1=7(7^k-1)+6=6(7m+1).**
4	Use induction to prove 3^n>=2n+1 for n>=0.	**Base: 3^0=1=2(0)+1. If 3^k>=2k+1, then $3^{(k+1)}$>=6k+3>=2k+3=2(k+1)+1.**
5	Use induction to prove 1+3+...+(2n-1)=n^2 for n>=1.	**Base: 1=1^2. If the sum through 2k-1 is k^2, adding 2(k+1)-1 gives k^2+2k+1=$(k+1)^2$.**

PART 2 · APPLY IT

1. **Total coins = 7n(n+1).** Base: stage 1 gives 14 coins. If the total through stage k is 7k(k+1), add 14(k+1) to get 7(k+1)(k+2).
2. **Total points = n(5n+1)/2.** Base: day 1 gives 3 points. If the total through day k is k(5k+1)/2, add 5(k+1)-2 to get (k+1)(5k+6)/2.

CHALLENGE

Base: 1^3=[1(2)/2]^2. Assume the sum through k is [k(k+1)/2]^2; adding $(k+1)^3$ gives $(k+1)^2$[k^2+4k+4]/4=[(k+1)(k+2)/2]^2.

Accept equivalent algebraic forms. For proof items, require a correct base case, inductive hypothesis, and step from k to k+1.